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Formal Dissipation-Dependent Effects of Nonlocality in the Electrodynamics of Surface Impedance for Conductors

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Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

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ABSTRACT

The study derives general frequency dependencies of the surface impedance modulus for conductors without the dc dissipation, i. e. for superconductors or perfect conductors. The frequency-dependent surface impedance was applied for the solutions corresponding to the spatially dispersive eigenvalues of the permittivity operator for conductors. The study demonstrates that appropriately taken into account effects of the spatial dispersion can give the general frequency dependence of the surface impedance for the obtained solutions including that for superconductors. It is shown that incorporation of the spatial dispersion leads to an appearance of the Meissner effect in perfect conductors in the same manner as in superconductors.

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1 INTRODUCTION

Recently the frequency-dependent surface impedance was calculated for the spatially dispersive eigenvalues of the permittivity operator in conductors [1]. It is reasonable to give a brief description of these formulations valid for both conductors and superconductors The electrodynamics of superconductors is supposed to be in principle reduced to those for conductors as the temperature approaches the critical temperature T_c and beyond T_c. However, such a reduction is not rather straightaway. Early the problem resulted in a supplement of the Maxwell equations by postulating additional London equations for an explanation the Meissner effect [2]. It discriminates the perfect conductor in non-zero-field cooling behaviour as compared to the superconductor. Here the study demonstrates that appropriately taken into account effects of the spatial dispersion can give general frequency dependencies of the surface impedance for the obtained in [1] solutions including those both for the normal conductor and for the superconductor or perfect conductor. It is shown that incorporation of the spatial dispersion leads to an appearance of the Meissner effect in perfect conductors in the same manner as in superconductors.

2 FREQUENCY DEPENDENCIES

To evaluate the general frequency dependence of the eigenvalue modulus $|\widetilde{\varepsilon}_a|$ of absolute permittivity operator $\widehat{\widetilde{\varepsilon}}_a$ and of the surface impedance modulus $|\widetilde{Z}|$, let us again [1] consider at first only the spatial effects in permittivity, assuming the problem is stationary $\omega = 0$ and taking into account only the spatial field inhomogeneity in the form of a wave number k''. Preliminary results concerning the frequency dependencies and spatial effects were discussed in [3, 4, 5].

2.1 Normal Conductivity

The first of Maxwell equations can be written as

$$\mathrm{rot}\,\widetilde{\mathbf{H}} = (\partial\widetilde{\mathbf{D}}/\partial t) + \widetilde{\mathbf{j}}. \qquad (2.1)$$

Below the vectorial values of electromagnetic field are meant as the eigenfunctions compliant with the eigenvalue of permittivity operator $\widehat{\widetilde{\varepsilon}}_a$, i. e. with the numerical complex value $\widetilde{\varepsilon}_a$ in the constitutive equation [1] $\widetilde{\mathbf{D}} = \widehat{\widetilde{\varepsilon}}_a\widetilde{\mathbf{E}} = \widetilde{\varepsilon}_a\widetilde{\mathbf{E}}$. The frequency $\omega''_k = k''v_F$ can be associated with the spatial inhomogeneity k'' [1], where v_F is the Fermi velocity, i. e. the velocity of propagation of the external perturbation. Using the ambiguity of the representation of a right side of the previous expression [6, 7, 8] and integrating it over the time one can get from the constitutive equation [1] $\widetilde{\mathbf{D}}(\mathbf{r}, t) = \widetilde{\varepsilon}_a\widetilde{\mathbf{E}}(\mathbf{r}, t)$ an expression [9]

$$\widetilde{\varepsilon}_a = \widetilde{\varepsilon}_p\varepsilon_0 - i\widetilde{\sigma}/\omega''_k = \widetilde{\varepsilon}_p\varepsilon_0 - i\widetilde{\sigma}/(k''v_F) \qquad (2.2)$$

Here $\widetilde{\sigma}$ is the static eigenvalue of conductivity operator, $\widetilde{\varepsilon}_p$ is the eigenvalue of relative permittivity operator of a lattice. The solutions found in Dresvyannikov et al. [1] for conductors and superconductors have the fixed arguments of complex numbers $\widetilde{\varepsilon}_a$ and $\widetilde{Z} \sim 1/\widetilde{k}$ determined by conditions of the spatial force resonances. So for a finite frequency ω of an external field the spatial inhomogeneity k'' is determined, according to Dresvyannikov et al. [1] and Karuzskii et al. [9], by the frequency and by the eigenvalue of permittivity operator

$$k'' \sim \omega(\widetilde{\varepsilon}_a\mu_0)^{1/2} = \omega/\widetilde{v}_{ph} \qquad (2.3)$$

That may be assigned to a mean field approximation. The conductivity in the extremely anomalous limit is determined by the frequency $\widetilde{\sigma} \sim 1/\omega$ also [10]. The substitution of k'' and $\widetilde{\sigma}$ into the expression (2.2) for $\widetilde{\varepsilon}_a$ results in the proportionality $(\widetilde{\varepsilon}_a)^{3/2} \sim \omega^{-2}$ that corresponds to the frequency dependence of the impedance modulus $|\widetilde{Z}| \sim \omega^{2/3}$ for all solutions found for the conductor except that for the superconductor, as it was shown in [1].

2.2 Superconductors and Perfect Conductors

To estimate the frequency dependencies of moduli $|\widetilde{Z}|$ and $|\widetilde{\varepsilon}_a|$ for the solution found for superconductors or perfect conductors having the phase of $\widetilde{\varepsilon}_a$ equal to $\beta_1 = \pi$ and the phase of $\widetilde{Z}$ equal to $\psi_1 = \pi/2$ (and additionally for the solution with $\alpha_0 = 0$ and $\varphi_0 = 0$) [1] one should note that for these solutions the real parts of eigenvalue $\widetilde{\sigma}$ of the conductivity operator in the equation (2.2) equal zero and the direct current does not dissipate with time. In this case one should take into account an influence of spatial inhomogeneities of the field with the largest possible value of a wave number k'' or of a frequency $\omega''_k = k'' v_F$, regarding an absence of the dc dissipation, for all possible spatial scales, but not only for those corresponding to the frequency of an incident wave (cf. the Eq. (2.3)). The velocity of propagation of a perturbation in the quasi-stationary environment without the dc dissipation could correspond to the spatially dispersive value v, which should not be necessarily associated with the Fermi velocity v_F, but may be equal to the speed of light c, for example, as in the Langmuir plasma frequency. The highest value of spatial inhomogeneity of the field with the largest possible value of a wave number k'' or of an associated frequency $\omega''_k = k'' v$, in this case, can approach the vertical-asymptote values equal to the k''_0 or to the frequency proportional to $\omega''_{kp0} = k''_0 v_F$, corresponding to the Langmuir plasma frequency

$$\omega_p = k''_0 c = c/\lambda_0, \tag{2.4}$$

where

$$\lambda_0 = (k''_0)^{-1} = (m/\mu_0 n e^2)^{1/2} \tag{2.5}$$

is the London penetration depth. The absence of the dc dissipation should result in the feasibility of "acoustic" conditions ($v > 0$) for the spatially dispersive plasmon polariton frequency $\omega''_{kp} = k'' v$.

This spatial inhomogeneity will correspond to the highest value of the Abraham force (see the Eq.(3.4) below and Ref. [1]). So it should result in the spatial structure dominating over the others possible structures [1, 9]. A value of this spatial inhomogeneity will not depend on frequency values ω of any incident microwaves, while those could be considered as quasi-stationary when $\omega < \omega''_{kp0} \ll \omega_p$. This can be illustrated by an expression of a current density after the end of an external perturbation obtained with the account of the spatial dispersion [10]. If one substitutes the spatial factor in Eq.(3) of Ref.[10] in the form of

$$j_0(\theta, r) = j_0(\theta) e^{i\kappa r}, \tag{2.6}$$

where θ is the reduced temperature $\theta = T/T_c$ [10, 11], it is reexpressed as the current density

$$j(\theta, r, t) = j_0(\theta) e^{-\omega'_k t} e^{i(\kappa r \pm \omega''_k t)}, \tag{2.7}$$

which remains after switching off an electric field $E = E_0 e^{i(kr+\omega t)}$. In a perfect conductor the direct current does not decay with time ($\omega'_k = 0$), and the stationary current solution (2.6) may be reconstructed from (2.7) by the proper time averaging of plasma oscillations with positive and negative signs of the plasmon-polariton frequency $\omega''_k = \omega''_{kp0}$, by a finite time shift in their starting moments, for example.

The requirement of an independence of the spatial inhomogeneity k'' on the frequency ω should result, according to the equation

$$k'' = \omega(|\widetilde{\varepsilon}_a|\mu_0)^{1/2} = \omega/|\widetilde{v}_{ph}|, \tag{2.8}$$

in the inverse ratio $\sqrt{|\widetilde{\varepsilon}_a|} \sim 1/\omega$ and in the dependence of the surface impedance (its modulus) linear on a frequency ω

$$|\widetilde{Z}| = \sqrt{\frac{\mu_0}{|\widetilde{\varepsilon}_a|}} = \frac{\omega\mu_0}{k''}. \tag{2.9}$$

It may look surprising that similar equations (2.3) and (2.8) produce different solutions. However, this is not an error, but those are two different possible formal solutions of this equation following from different conditions of spatial dispersion. The frequency dependence of the eigenvalue $\widetilde{\varepsilon}_a$ in the right side of the Eq. (2.8) is determined by the requirement of the independent on ω wavenumber k'' in the left side, inversely to their correlations in the Eq. (2.3), where the wavenumber k'' is determined by the eigenvalue $\widetilde{\varepsilon}_a$ according to the mean-field approximation. The further temperature issue of how the electrodynamics of superconductors can be reduced to those for conductors as the temperature approaches and beyond the critical temperature may be clearly stated using, e. g., the generalised two-fluid principles [10, 11] or rigorous modelling [11, 12].

3 THE SPATIAL DISPERSION AND MEISSNER EFFECT IN SUPERCONDUCTORS AND PERFECT CONDUCTORS

In the surface impedance approximation [6, 7, 8, 9, 11, 12] let it will be the coordinate z with the basis vector $\mathbf{k}$, which are directed into the conductor together with the unitary normal $\mathbf{n}$ to the boundary surface $z = 0$. A monochromatic transverse field at the normal incidence to the plane surface of a conductor will be considered. An electric field strength vector is directed tangentially along the x-axis ($\widetilde{\mathbf{E}}(z) = \widetilde{E}_x(z)\mathbf{i}$), and a magnetic induction vector is directed along the y-axis ($\widetilde{\mathbf{B}}(z) = \widetilde{B}_y(z)\mathbf{j}$). Here **i** and **j** are the basis vectors. The fields decrease into the depth of a conductor as $\widetilde{B}_y(z) = \widetilde{B}_y(0)e^{-z/\widetilde{\delta}}$ and $\widetilde{E}_x(z) = \widetilde{E}_x(0)e^{-z/\widetilde{\delta}}$, where $\widetilde{\delta}$ is the complex penetration depth, which defines the surface impedance by the relation $\widetilde{Z} = i\omega\mu_0\widetilde{\delta}$ [6, 7, 8, 9, 11, 12], and μ_0 is the permeability of vacuum. Since the surface impedance relates the tangential components of fields on the boundary surface [9, 11]

$$\widetilde{Z} = \frac{\widetilde{E}_x(0)}{\widetilde{H}_y(0)} = \frac{\widetilde{B}_y(0)}{\widetilde{D}_x(0)}, \qquad (3.1)$$

the zero limit of the surface impedance $\widetilde{Z}|_{\omega\to 0} = 0$ at the zero frequency, deduced from the Eq. (2.9), means that the stationary tangential component of an electric field (or of a magnetic induction) should be equal zero $\widetilde{E}_x(0)|_{\omega\to 0} = 0$ (or $\widetilde{B}_y(0)|_{\omega\to 0} = 0$), but with a possible finite value of $\widetilde{H}_y(0)|_{\omega\to 0} \neq 0$ (or $\widetilde{D}_x(0)|_{\omega\to 0} \neq 0$). An alternative combination of a finite value of $\widetilde{B}_y(0)|_{\omega\to 0}$ with an infinite $\widetilde{D}_x(0)|_{\omega\to 0}$ is possible in situations when the direct current flows without dissipations. Formally the Eq. (3.1) in this case, corresponds to the indeterminate form such as $0 \cdot \infty$, an evaluation of which is closely related to the Meissner effect. The remainder of this study is devoted to the problem of evaluating this indeterminate form by substituting it in the expression of Abraham force.

Let us consider a behaviour of the Abraham force [6, 8] in a perfect conductor (or in a superconductor) in the limit of zero frequency $\omega \to 0$. From the expression for the spatial density of the momentum flux [1, 9], which is the corresponding component of the Maxwellian field-stress tensor,

$$\widetilde{\Pi} = [\widetilde{\mathbf{D}} \times \widetilde{\mathbf{B}}] = (\mu_0^{1/2}\widetilde{\varepsilon}_a^{3/2})\widetilde{E}_x^2(0)\mathbf{k}, \qquad (3.2)$$

where $\mathbf{k}$ is the basis vector of the z-axis, the Abraham force can be derived [6, 8] neglecting the frequency dispersion:

$$\begin{aligned}
&\left(1 - \frac{1}{\widetilde{\widetilde{\varepsilon}}}\right)\frac{\partial\widetilde{\Pi}}{\partial t} = \left(1 - \frac{1}{\widetilde{\widetilde{\varepsilon}}}\right)\frac{\partial}{\partial t}[\widetilde{\mathbf{D}} \times \widetilde{\mathbf{B}}] = \\
&= \left(1 - \frac{1}{\widetilde{\widetilde{\varepsilon}}}\right)\frac{\partial}{\partial t}\left[\widetilde{D}_x(0)e^{i\omega t}\mathbf{i} \times \widetilde{B}_y(0)e^{i\omega t}\mathbf{j}\right] = \\
&= 2i\omega\left(1 - \frac{1}{\widetilde{\widetilde{\varepsilon}}}\right)e^{i2\omega t}\left[\widetilde{D}_x(0)\mathbf{i} \times \widetilde{B}_y(0)\mathbf{j}\right],
\end{aligned} \qquad (3.3)$$

where $\widetilde{\varepsilon} = \widetilde{\varepsilon}_a/\varepsilon_0$ is the eigenvalue of relative permittivity operator. The surface impedance (3.1) defines the relation between electric and magnetic inductions $\widetilde{D}_x(0)$ and $\widetilde{B}_y(0)$ and may be substituted in the expression (3.3) for the Abraham force. Then using the Eq. (2.9) for the modulus of surface impedance and taking account of its phase $\psi_1 = \pi/2$

$$\begin{aligned}
&2i\omega\left(1 - \frac{1}{\widetilde{\widetilde{\varepsilon}}}\right)e^{i2\omega t}\left[\frac{\widetilde{B}_y(0)}{\widetilde{Z}}\mathbf{i} \times \widetilde{B}_y(0)\mathbf{j}\right] = \\
&= 2i\omega\left(1 - \frac{1}{\widetilde{\widetilde{\varepsilon}}}\right)e^{i2\omega t}\left[\frac{-ik''_A}{\omega\mu_0}\widetilde{B}_y(0)\mathbf{i} \times \widetilde{B}_y(0)\mathbf{j}\right] = \\
&= 2\left(1 - \frac{1}{\widetilde{\widetilde{\varepsilon}}}\right)e^{i2\omega t}\left[\frac{k''_A}{\mu_0}\widetilde{B}_y(0)\mathbf{i} \times \widetilde{B}_y(0)\mathbf{j}\right] = \\
&= 2\left(1 - \frac{1}{\widetilde{\widetilde{\varepsilon}}}\right)e^{i2\omega t}\mu_0\left[\mathrm{rot}\widetilde{\mathbf{H}}(0) \times \widetilde{\mathbf{H}}(0)\right].
\end{aligned} \qquad (3.4)$$

The expression (3.4) shows that Abraham force at all frequencies $\omega \ll \omega_p$, as well as in the stationary limit $\omega \to 0$, achieves the highest magnitude when the spatial inhomogeneity k''_A gets the largest, frequency-independent value (see Sec. 2.2) corresponding to the London (or plasma) penetration depth $\lambda_0 = (k''_0)^{-1} = (m/\mu_0 ne^2)^{1/2}$. Such the spatial configuration of the fields related to the reconstruction of an electronic system, which corresponds to the highest magnitude of the Abraham force, is dominant due to the force correlations and the consequential decrease of the free energy [1, 9]. And respectively, the absolutely unstable spatial configuration, according to the Eq. (3.4), corresponds to the geometry of the spatially homogeneous field and the zero value of the spatial inhomogeneity $k''_A = 0$ when the Abraham force is zero with no advantage in the free energy. So the conclusions obtained here in the surface impedance approximation show that the spatial dispersion results in an appearance of the Meissner effect in perfect conductors in the same manner as in superconductors contrary to the preceding considerations [2], which do not incorporate the spatial dispersion effects. A driving force of its appearance is the Abraham force.

It should be noted that this conclusion may be derived more formally, without considerations of any frequency dependencies. Abraham force can be reexpressed as

$$\left(1-\frac{1}{\widetilde{\varepsilon}}\right)\frac{\partial\widetilde{\Pi}}{\partial t}=\left(1-\frac{1}{\widetilde{\varepsilon}}\right)\frac{\partial}{\partial t}\left[\widetilde{\mathbf{D}}\times\widetilde{\mathbf{B}}\right]=$$
$$=\left(1-\frac{1}{\widetilde{\varepsilon}}\right)\left[\frac{\partial\widetilde{\mathbf{D}}}{\partial t}\times\widetilde{\mathbf{B}}\right]+\left(1-\frac{1}{\widetilde{\varepsilon}}\right)\left[\widetilde{\mathbf{D}}\times\frac{\partial\widetilde{\mathbf{B}}}{\partial t}\right] \tag{3.5}$$

For monochromatic fields the Maxwell equations in a homogeneous medium with the spatial dispersion may be written [1, 6, 7, 8, 9] as

$$\mathrm{rot}\widetilde{\mathbf{B}}=\mu_0\mathrm{rot}\widetilde{\mathbf{H}}=\mu_0(\partial\widetilde{\mathbf{D}}/\partial t);\quad \mathrm{rot}\widetilde{\mathbf{E}}=-(\partial\widetilde{\mathbf{B}}/\partial t); \tag{3.6}$$

$$\mathrm{div}\widetilde{\mathbf{D}}=\mathrm{div}\widetilde{\mathbf{E}}=0;\qquad\qquad \mathrm{div}\widetilde{\mathbf{B}}=0. \tag{3.7}$$

Due to an absence of magnetic charges and an equality $\partial\widetilde{\mathbf{B}}/\partial t = 0$ in the second of Eqs (3.6) in the stationary approximation $\omega = 0$ the second term in the sum (3.5) equals zero. Regarding the ambiguity of the representation of the first Maxwell equation (cf. the first of Eqs (3.6) and the Eq.(2.1) $\mathrm{rot}\ \widetilde{\mathbf{H}} = (\partial\widetilde{\mathbf{D}}/\partial t) + \widetilde{\mathbf{j}}$), one can replace the partial time derivative in the first term of the sum (3.5), which may be assigned formally to the "stationary displacement current" or simply to the current density $\widetilde{\mathbf{j}}$, by the $\mathrm{rot}\widetilde{\mathbf{H}}$ from the first of Eqs (3.6), that results in

$$\left(1-\frac{1}{\widetilde{\varepsilon}}\right)\left[\mathrm{rot}\widetilde{\mathbf{H}}\times\widetilde{\mathbf{B}}\right]=\left(1-\frac{1}{\widetilde{\varepsilon}}\right)\left[\frac{k''_{ef}}{\mu_0}\widetilde{B}_y(0)\mathbf{i}\times\widetilde{B}_y(0)\mathbf{j}\right]. \tag{3.8}$$

This expression shows that Abraham force achieves the highest magnitude when the spatial inhomogeneity k''_{ef} gets the largest value. It corresponds to a half of the London penetration depth $\lambda_0/2 = (2k''_0)^{-1} = (m/\mu_0 n4e^2)^{1/2}$ taking into account a factor of 2 that differentiates the Eqs (3.4) and (3.8) in the stationary approximation $\omega = 0$.

4 FORMAL BACKGROUNDS OF THE ELECTRODYNAMIC NONLOCALITY

To find the origin of this double ratio formal backgrounds of the electrodynamic nonlocality is deduced in this section with an emphasis on the analysis of nonself-adjoint problems. Since it may be not quite comprehensible for general interest readers with a background in the condensed matter due to the presence of rather abstract technical formulations, this section can be skipped in first reading.

The double ratio $k''_{ef} = 2k''_A$ in the force expressions can be understood considering that the operator $\mathrm{rot}\widetilde{\mathbf{H}}$ in the Eq.(3.8) acts for the "stationary displacement current", i. e. for the displacement-current operator $(\partial\widetilde{\mathbf{D}}/\partial t)$ in the zero frequency limit. So it should resemble fundamental properties of a displacement current. Nonlocality is its basic property that comes into definitions of electric and magnetic polarisations and inductions via the spatially dependent electric and magnetic moments. In the continuous media electrodynamics [6, 7, 8] these moments are introduced as the electric or magnetic moment formed on the boundary surface of the body and related to a geometric shape of the surface. The boundary surface usually is approximated by the certain quadratic form [13] of vectors in a three-dimensional linear space, which is a mathematical abstraction of our empirical coordinate space with the real-valued Euclidean square. This linear space is a linear manifold spanned by the three real spatial basis vectors [13] with a basic reference standard of the length defined materially. It may be considered as the space V_3 the elements of which are vectors (directed line segments) subject to certain suitably defined operations. These vectors are studied in three-dimensional analytic geometry and mechanics. V_3 is a linear space over the field R of real numbers [13]. This mathematical abstraction allows describing empiric physical quantities numerically as vector or scalar fields in that, given by experiences, real-valued Euclidean coordinate space with three real spatial basis vectors. Being defined in this abstract linear space V_3 the scalar, vectorial and mixed products of vectors are assumed to be applicable as well to the vectors of the fields of physical quantities as if those belong to the linear space. Furthermore, in differential operators of vector or scalar fields of physical quantities, such as Hamilton or Laplace operators, it is these three-dimensional real spatial variables, concerning those the spatial derivatives are taken. One can remember differential operators in the equations of Maxwell (3.6), (3.7), or of Newton with a gradient of a scalar field potential, vector fields of electric magnitudes or of velocities, scalar fields of spatial densities of an electric charge or of a mass. The assumption of applicability of the products of three-dimensional vectors and of the differential field operators to the vector manifold of empiric physical fields as if it belongs to the space V_3 allows using those in the equations, which explain the physical phenomena.

For this abstract boundary surface to be an electrodynamics boundary surface, i. e. to produce any observable physical effect, it should separate the bounded regions of the space by different values of at least the one more real-valued variable parameter, which may vary independently of variations of the three spatial coordinates. It corresponds to the inhomogeneous distribution of this parameter over the space V_3. In Maxwell equations, such novel independent variables are the constitutive parameters or a combination of these parameters, which characterise material properties. Those are the specific bulk densities of a charge and of a mass as well as the absolute permittivity, which is a combination of these densities and of a frequency, i. e. of the time-related parameter. The absolute permittivity (cf. [6, 7, 8, 10, 11] and Eq. (2.2)) $\widetilde{\varepsilon}_a = \widetilde{\varepsilon}_p\varepsilon_0 - i\widetilde{\sigma}/\omega = \widetilde{\varepsilon}_p\varepsilon_0 - i\varepsilon_0\omega_p^2/[\omega(\widetilde{\omega}_k + i\omega)] = \widetilde{\varepsilon}_p\varepsilon_0 - i/[\lambda_0^2\mu_0\omega(\widetilde{\omega}_k + i\omega)] = \widetilde{\varepsilon}_p\varepsilon_0 - ine^2/[m\omega(\widetilde{\omega}_k + i\omega)] = \widetilde{\varepsilon}_p\varepsilon_0 - i(ne)^2/[(nm)\omega(\widetilde{\omega}_k + i\omega)]$ depends on the charge (ne) and mass (nm) densities, on the time $(1/\omega)$, and combines the basic units of length, mass, time and electric charge.

These additional real-valued variables of mass, time and charge are independent of variations of the three spatial length coordinates and consequently should result in the greater than 3 dimension of the abstract linear space over the field R of real numbers [13] to proceed the description of empiric physical fields as the vectors existing in a linear space, but with a number of real-valued coordinates greater than 3.

The linear description allows using advantages of finite-dimensional linear spaces [13] given by the application of linear operations in most powerful techniques.

The increased dimension requires an appearance of the respective number of additional basis vectors, which are linear-independent of three real spatial basis vectors of the three-dimensional linear space V_3 comprising a linear manifold spanned by these three real basis vectors. Since the given by experiences dimension of our coordinate space is limited by three real spatial basis vectors, e. g. $\mathbf{i}, \mathbf{j}, \mathbf{k}$, the additional basis vectors must be *notional*, not existing in our three-dimensional space, e. g. $i\mathbf{i}$, $i\mathbf{j}$, $i\mathbf{k}$ [13], because in the linear manifold spanned by these three real basis vectors there is no vacant space for the fourth or higher linear-independent *real* vector. The variables of mass, time and charge are meant as real-valued so the complex manifold of vectors of physical fields with additional imaginary basis vectors should be considered as vectors of the real linear space $\mathbf{C}_6$ over the field R of real numbers [13] with the real pseudo-euclidean square [14] that results from the mutual orthogonality of the sextet of all real and imaginary basis vectors. The additional basis notional vectors $i\mathbf{i}$, $i\mathbf{j}$, $i\mathbf{k}$ have in this real pseudo-euclidean space the pure imaginary unitary value of the "length" [14, 15], which is derived from the pseudo-euclidean square defined in the same manner as in the real Euclidean space V_3 and agrees with the requirement of nonexistence of these notional vectors in the space V_3.

The similar linear space has been considered formally in more details [1] when seeking a solution of the nonself-adjoint Laplace operator derived from Maxwell equations, which corresponds to the eigenvalue in the form of the complex Euclidean square of some vector $\widetilde{\mathbf{k}}$ in the real-base three-dimensional Euclidean space, but generally with complex components. This vector – the wave vector may be considered as a vector of the three-dimensional complex Euclidean space [14] $\mathbf{C}_3$ over the field C of complex numbers [13]. The similarly specified complex n-dimensional space was introduced in finding a modification of the Jordan matrix suitable for the case of a real space [13]. In the three-dimensional complex Euclidean space the scalar, vectorial and mixed products of vectors may be considered formally, at least in the component form, as defined in the real Euclidean space V_3. However, the derivatives in differential operators are taken with respect to the real spatial variables [1].

Certain general geometric restrictions have been noted [1] concerning a behaviour of vectors from the complex Euclidean space if those are considered in the real three-dimensional space, which is associated with the physical reality. The restrictions relate to a known duality [13, 14] of the representation of the one and the same complex affine space as the spaces of different dimension over the fields of complex (e. g. $\mathbf{C}_3$ over the field C) or real (e. g. $\mathbf{C}_6$ over the field R) numbers. Subscripts denote here the number of basis vectors. This duality is closely related to the fundamental theorem of algebra, to polynomial factoring with coefficients in the field R of real numbers and to the real Jordan canonical form [13]. In general, there is no canonical basis in which the matrix of a linear operator acting in a real *n*-dimensional space $\mathbf{R}_n$ takes the Jordan form [13], if only because the characteristic polynomial of the operator can have imaginary roots. To find a modification of the Jordan matrix suitable for the case of a real space the linear transformation is used which constructs a basis in the real space $\mathbf{R}_n$ by replacing each pair of the complex conjugate vectors of the Jordan basis by a pair of real vectors [13]. The transformation resembles the similar one known in theories of superfluidity and superconductivity as Bogoliubov transformation. The different dimensionality of complex and real representations of the unique complex affine space results in different possible definitions (see Sections 8.2, 9.2, 9.4 in [13]) of the products of vectors in these complex affine spaces, including the mentioned above pseudo- and complex Euclidean square [14]. That ambiguity could expand [1] the variety of possible solutions of the non-self-adjoint problems and might occur to be useful in such the first, linear approach to finding solutions and to the future analysis of obtained solutions, which in general should be verified empirically, however.

In the second, nonlinear approach the restrictions are a result of a consideration of the vectors from the real linear space $\mathbf{C}_6$ over the field R of real numbers, which is the linear manifold spanned by the sextet of real and imaginary basis vectors and has an even dimensionality as the real number representation of the complex affine space [13, 14]. To the contrary, we consider such vectors as the vectors of the fields of physical quantities as if those reveal themselves in the three-dimensional space V_3 over the field R of real numbers with the real original basis $\mathbf{i}, \mathbf{j}, \mathbf{k}$, a dimension of which is not even. As a result, this three-dimensional real space complemented by the complex vector manifold of physical fields cannot be in general a three-dimensional linear space due to its odd dimensionality. Such a three-dimensional combination of the space and the manifold cannot be the real number representation of the complex *affine* space, the dimensionality of which should be even [13, 14]. Consequently this combination can not be a subspace of the complex space $\mathbf{C}_3$ over the field C or of the real space $\mathbf{C}_6$ over the field R. It may form only [1] a three-dimensional factor space [13] of the $\mathbf{C}_6$ over the field R with respect to the kernel subspace consisting, for example, of the pure imaginary vectors of this real space $\mathbf{C}_6$, which transforms into the null-vector of the real-base three-dimensional space V_3 over the field R of real numbers.

An injection of the space V_3 by the vector manifold of the empiric physical fields allows to use the suitably defined and verified empirically operations of products of three-dimensional vectors and the differential field operators in expressions, resembling empiric physical laws, for all vectors of the manifold as if it belongs to the space V_3. However, in this second approach the space V_3 can not continue to be the affine space being combined with the manifold. It means that the "physical space", comprising the combined vector manifolds of the space V_3 and of the empiric physical fields, loses main attributes of the *linear three-dimensional* space over the field R of real numbers, which include the addition rule of vectors and the rule for multiplication of a vector by a number together with respective axioms obeyed by these operations [13]. The vector manifold of empiric physical fields, being combined with the space V_3, transforms it into the *nonlinear three-dimensional* vector manifold corresponding to the three-dimensional factor space [13] of the $\mathbf{C}_6$ over the field R. Consequently there is no real three-dimensional basis which could produce this manifold as a linear manifold over the field R of real numbers spanned by three vectors of this real basis. It means that such basis could not be obtained by any linear mapping of the original real three-dimensional basis $\mathbf{i}, \mathbf{j}, \mathbf{k}$ of the space V_3 which would be described by an 3×3 matrix of real constants. Otherwise, the existence of a linear transformation, which could produce such basis, should lead to the linearity of a vector manifold of the "physical space" due to the general K-isomorphism of all n-dimensional (here $n = 3$) linear spaces over a field K [13].

Thus a linear mapping of V_3 should be replaced by a general nonlinear transformation to describe a vector manifold of the "physical space". Such transformation may be thought as an injection of the additional vector manifold of physical fields into the linear space V_3 accompanied by the tensile-compressive deformation of the last. It can be described by mapping the three-dimensional Euclidean linear space V_3 into its three-dimensional "frames" occupied by the combined Euclidean vector manifold of this former space V_3 and of physical fields and can be defined, for example, by an explicit real function $\mathbf{y} = \mathbf{y}(\mathbf{x})$, where $\mathbf{x} = (x_1, x_2, x_3)$ and $\mathbf{y} = (y_1, y_2, y_3)$ are vectors of the space V_3 and of the combined "physical" manifold respectively, by a more general implicit relation $F(\mathbf{x}, \mathbf{y}) = 0$, or by a parametrically definable function.

In accordance with [15] let $d\mathbf{y}\, d\mathbf{y}$ and $d\mathbf{x}\, d\mathbf{x}$ be Euclidean squares of the "distance" between the points (vectors) $\mathbf{y},\ \mathbf{y} + d\mathbf{y}$ and $\mathbf{x},\ \mathbf{x} + d\mathbf{x}$. Those are the comparable line elements in the nonlinear vector manifold and in the linear space V_3, from which this nonlinear manifold has been transformed. Let $d\sigma^2$ and $d\mathbf{s}^2$ denote the Euclidean squares, respectively. The mapping is Gauss conformal if there is a positive scalar function $\lambda(\mathbf{x})$ that satisfies the relation $d\sigma^2 = \lambda(\mathbf{x}) d\mathbf{s}^2$. It is angle-preserving and Liouville's theorem on conformal mappings in Euclidean space states [15] that any smooth conformal mapping on a domain of $\mathbf{R}_n$, where $n > 2$, can be expressed as a composition of a finite number of translations, similarities, orthogonal transformations and inversions: they are Möbius transformations (in n dimensions). This theorem severely limits the variety of possible conformal mappings in $\mathbf{R}_3$ (here in V_3), and higher-dimensional spaces.

Three of four possible elementary operations of translations

$$\mathbf{y} = \mathbf{x} + \mathbf{h}\,, \tag{4.1}$$

similarities $\mathbf{y} = \mu\mathbf{x}$, orthogonal transformations $\mathbf{y} = C\mathbf{x}$, where $\mathbf{h} = (h_1, h_2, h_3)$ is the constant vector, μ - the real constant scalar, and C - the orthogonal matrix [13, 15] of real constants, are the linear maps with $\lambda = 1$ for the first and third operations and with $\lambda = \mu^2$ for the second operation [15]. To ensure the inevitable nonlinearity of a vector manifold of the "physical space" these conformal maps of translations, similarities, orthogonal transformations or any composition comprising only these elementary linear maps should be eliminated from possible general transformations of V_3 into the "physical space". Conformal maps of translations and similarities and of orthogonal transformations may be considered, respectively, as global translational (translational-dilatational, in general) and rotational symmetries of a finite-dimensional affine space. It means that these global symmetries correspond to linear maps of the affine space into itself. Consequently, the physical nonlocality, which is embedded in Maxwell equations and has been discussed in the beginning of this section, leads to the inevitable nonlinearity of the vector manifold of the non-empty "physical space" resulting in the broken global translational and rotational symmetries of the formerly empty affine space V_3. These global symmetries may be considered as necessary and sufficient conditions of the linearity of a real finite-dimensional vector space (in n dimensions, where $n > 2$), a failure of which corresponds reasonably to the supposed above inhomogeneous distribution of constitutive parameters over the space V_3. Necessity follows from the definition [13] of linear space and sufficiency has been proven just above.

Thus the only allowed one of four possible in V_3 elementary operations of global Gauss conformal maps is the inversion

$$\mathbf{y} = \frac{\mathbf{x}}{(\mathbf{x}\cdot\mathbf{x})} \tag{4.2}$$

of the affine space V_3 into a nonlinear vector manifold of the "physical space" with the nonlinear scalar function $\lambda(\mathbf{x}) = (\mathbf{x}\cdot\mathbf{x})^{-2}$ [15], where $(\mathbf{x}\cdot\mathbf{x})$ is the scalar product. Any arbitrary composition of elementary conformal maps of the space V_3 into the nonlinear manifold of "physical space", which may be required in the solution of an electrodynamic problem, should necessarily incorporate the nonlinear map of inversion among four possible types of conformal maps. This conclusion illustrates the mentioned above tensile-compressive deformation of the "physical space". It is a consequence of the duality in the representation of the complex affine space as the complex or real space and finds good agreement with the conception of the phase space.

The ambiguity in possible definitions of products of vectors in finite-dimensional complex affine spaces can expand, as it has been previously discussed in this section, the variety of possible solutions of the non-self-adjoint problems, if only because the adjoint operators are defined concerning the fixed bilinear form [13, 14]. This ambiguity can lead also to different nonlinear metric properties of a vector manifold of the "physical space". It may be illustrated by the simplest one-dimensional complex space $\mathbf{C}_1$ and by its two-dimensional real number representation $\mathbf{C}_2$. Let $d\mathbf{z}\,d\mathbf{z}$ be again a scalar square of the "distance" between the complex points (complex vectors) $\mathbf{z}$, $\mathbf{z}+d\mathbf{z}$, i. e. of the line element, where $\mathbf{z} = (x+iy)\mathbf{i}$ and $\mathbf{z}+d\mathbf{z} = ((x+dx)+i(y+dy))\mathbf{i}$ are vectors of the complex space $\mathbf{C}_1$. Let $\mathbf{C}_1$ be a complex unitary space with the Hermitian scalar product [13] of the line element $d\mathbf{z}\,d\mathbf{z} = ((dx+idy)\,\mathbf{i},\,(dx-idy)\,\mathbf{i}) = dx^2+dy^2$ or, alternatively, be a complex Euclidean space [14] with the Euclidean scalar square $d\mathbf{z}\,d\mathbf{z} = ((dx+idy)\,\mathbf{i},\,(dx+idy)\,\mathbf{i}) = (dx^2-dy^2)+2\,i\,dx\,dy$. Consider the respective real space $\mathbf{C}_2$ consisting of the formal sums $\mathbf{z} = x\ \mathbf{i} + y(i\ \mathbf{i})$ and $\mathbf{z}+d\mathbf{z} = (x+dx)\ \mathbf{i} + (y+dy)(i\ \mathbf{i})$, which are vectors in a complex orthonormal basis of the real space $\mathbf{C}_2$ [13] (taking account of the pure imaginary unitary "length" of the basis notional vector). The following natural scalar products may be defined arbitrarily, being not predetermined a priori as well as for the space $\mathbf{C}_1$, by the formulae $d\mathbf{z}\,d\mathbf{z} = ((dx\ \mathbf{i} + dy(i\,\mathbf{i})),(dx\ \mathbf{i} + dy(-i\,\mathbf{i}))) = dx^2+dy^2$ corresponding to the real Euclidean space with the Hermitian scalar product of the basis notional vector, or alternatively, by $d\mathbf{z}\,d\mathbf{z} = ((dx\ \mathbf{i} + dy(i\,\mathbf{i})),(dx\ \mathbf{i} + dy(i\,\mathbf{i}))) = dx^2 - dy^2$ corresponding to the real pseudo-euclidean space [14] with the Euclidean scalar square of the basis notional vector.

A general nonlinear transformation into a vector manifold of the "physical space" for the case of $\mathbf{C}_2$ can be described by mapping the one-dimensional Euclidean linear space V_1 into its one-dimensional "frames" occupied by the combined vector manifold of this former space V_1 and of physical fields and can be defined, for example, by a parametrically definable function using the component x of the real basis vector as a parameter, i. e. $\mathbf{z}(x) = x\,\mathbf{i} + y(x)(i\,\mathbf{i})$. It allows to estimate a respective transformation of a scalar square of the line element. Thus for considered above scalar products of the line element $d\sigma^2$ one obtains the following x-dependencies $d\sigma^2 = \lambda(x)d\mathbf{s}^2 : [1 + (dy/dx)^2]dx^2$ for a complex unitary space $\mathbf{C}_1$ with the Hermitian product as well as for a real Euclidean space $\mathbf{C}_2$ with the Hermitian product of the basis notional vector, $[1 + i(dy/dx)]^2dx^2 = [1 - (dy/dx)^2 + 2i(dy/dx)]dx^2$ for a complex Euclidean space [14] $\mathbf{C}_1$ with the Euclidean scalar square, and $[1 - (dy/dx)^2]dx^2$ for a real pseudo-euclidean space [14] $\mathbf{C}_2$ with the Euclidean scalar square of the basis notional vector. A scalar function $\lambda(x)$ characterising a transformation of the metric is generally the nonlinear square-law real function, except the complex function for the complex Euclidean space $\mathbf{C}_1$. This transformation function is of an elliptic type for the spaces $\mathbf{C}_1$ and $\mathbf{C}_2$ with the Hermitian product, and of a hyperbolic type for the spaces $\mathbf{C}_1$ (at least for the real part of $\lambda(x)$) and $\mathbf{C}_2$ with the Euclidean scalar square. In this last case of the pseudo-euclidean space $\mathbf{C}_2$ there is a limitation of the value of derivative in the inequation $\lambda(x) = [1 - (dy/dx)^2] \geq 0$ to provide the real, not pure imaginary length of the line element for it would not belong to the zero class of a factor space and could be measurable in the "physical space" of real vectors.

5 EFFECTIVE PARAMETERS AND ABRAHAM FORCE

The nonlocality may be considered as an indication of inhomogeneity of the "physical space" corresponding to the broken global translational symmetry, that has been formally demonstrated in the previous section 4. In the problem considered here the nonlocality is revealed as a requirement of electroneutrality, i. e. as a requirement of an absence of extrinsic charges (see the first of Eqs (3.7)). The requirement of electroneutrality results in the zero values of the bulk integral electric charge and of the total kinetic (true) momentum $\boldsymbol{p} = 0$ in the absence of extrinsic currents, if the total momentum can be written in the form characteristic of the rigid medium $\boldsymbol{p} = m\boldsymbol{v}$ where the velocity $\boldsymbol{v}$ is the same constant vector for all particles or parts of the rigid medium. Here the velocity $\boldsymbol{v}$ of an intrinsic charge carrier is measured relative to the rest reference coordinate system which is exterior to the conductor, m is the parameter expressed in units of mass. The approximation of a rigid medium correlates with the discussed in section 2.2 feasibility of "acoustic" conditions for the spatially dispersive plasmon polariton frequency and with Landau criterion for superfluidity. The presence of the critical minimal velocity for intrinsic elementary excitations in the system leads to the movement of the system as a whole, i. e. as a rigid medium when the velocity of the movement is lower than the critical velocity for intrinsic excitations.

The latter requirement of the zero total momentum just corresponds to inhomogeneity of the space relative to the movement of participate matter considered as a rigid medium. Otherwise, the constant non-zero vector of the total true (kinetic) momentum $\boldsymbol{p} = m\boldsymbol{v} \neq 0$ would correspond to the invariant state of the "physical space" comprising current carriers. The instant states of such physical system could only differ in just spatial coordinates of current carriers. These variations of coordinates would not affect all other parameters of the momentum-conserved state. Thus coordinates of current carriers in every instant state of such physical system could be linearly transformed by translations of the space V_3, in which the coordinates should be determined. The constant vector in the linear map of translations (4.1) would be equal to $\mathbf{h} = \boldsymbol{p}\Delta t/m = \boldsymbol{v}\Delta t$, where Δt should be the respective interval of time. However, the linear conformal map of translations (cf. Eq. (4.1)) must be excluded from possible general transformations of a finite-dimensional affine space V_3 into the nonlinear vector manifold of the "physical space" as it has been formally derived in section 4. Consequently, the global translational symmetry of a finite-dimensional affine space should be broken in the nonlinear real vector manifold of the "physical space" where coordinates of current carriers would be determined. The non-zero vector of the total true (kinetic) momentum $\boldsymbol{p} \neq 0$ can not belong to this "coordinate physical space" in the approximation of rigid medium. Formal backgrounds of the previous section allow concluding that non-zero vectors of the total true (kinetic) momentum can belong only to the zero class of a factor space, have the pure imaginary "length" and cannot be measurable in the real "coordinate physical space" in this approximation.

These conclusions cannot be generalised with respect the composite non-rigid medium of current carriers even under conditions of electroneutrality since the mass factor, which has been additive in the rigid-medium approximation $m = \sum m_i$, comes in the linear combination with possibly different vectors of velocity and cannot be involved reciprocally in the vector of translation $\mathbf{h}$. As a result the map of translations (cf. Eq. (4.1)) for the composite non-rigid medium fails to be linear on the kinetic momentum ($\mathbf{h} \sim \boldsymbol{p}$) in general. This different behaviour of rigid and composite non-rigid media correlates with earlier discussions of electrodynamic nonlocality (cf. [16] and references therein).

A displacement current is a movement of the intrinsic charge carriers and can possess only the quasi-momentum $\hbar\boldsymbol{k} \neq 0$ in the rigid-medium approximation. The quasi-momentum vector manifold corresponds to the allowed conformal map of inversions (cf. Eq. (4.2) and discussions in section 4).

A stationary current density $\widetilde{\mathbf{j}}$, substituted into the Eq. (3.8) instead of the $(\partial\widetilde{\mathbf{D}}/\partial t)$ in the form of $\mathrm{rot}\widetilde{\mathbf{H}}$, may not satisfy the basic requirement of electroneutrality being generally composed of the induced intrinsic and extrinsic components and can have the total kinetic (true) momentum $\boldsymbol{p} \neq 0$ even in the rigid-medium approximation.

To fulfil the electroneutrality requirement unambiguously one can introduce it, together with the condition $\boldsymbol{p} = 0$, in the constitutive parameter k''_{ef} in the Eq.(3.8) similar to the manner used in the quasiparticle formalism. A displacement current alters an electric dipole moment, which is defined as an electric charge multiplied by a distance $q \cdot \Delta x$. Let an increment of the electric dipole moment in the first interior coordinate system, acquired by a moving charge in a time interval t_0, is equal to $q_m \Delta x_m$ where $\Delta x_m = v_m t_0$ and v_m is the velocity of the charge movement in this first coordinate system, relative to the x-axis of which a charge is moving. Since in the surface impedance (plane wave) approximation the three-dimensional electrodynamics problem is reduced to the one-dimensional partial differential equation [6, 7, 8, 9, 11], one can use the specific linear density of the mobile charges η in place of the specific bulk charge density generally considered [6]. In a conductor, it should be compensated by the same value of the specific linear density of the charge of the opposite sign ρ to fulfil the electroneutrality requirement. It may be treated as a charge of the background and can be assigned to the interior coordinate system. In this paradigm the moving intrinsic charge may be assumed as an integrator of the incoming specific linear density of mobile charges in the process of a charge motion along the x-coordinate, e. g., $q = \eta \cdot |\Delta x|$. The dipole moment, acquired by a charge moving relative to the first interior coordinate system $q_m \Delta x_m$ where $\Delta x_m = v_m t_0$, due to an absence of extrinsic charges should be equal to the dipole moment acquired by a charge in any interior coordinate system as well as in the second interior coordinate system, relative to which the charge looks like immobile in the sense of the zero kinetic (true) momentum in the exterior rest coordinates, $q_L \Delta x_L$ where $\Delta x_L = v_L t_0$ and v_L is the velocity of the charge movement in this second interior coordinate system. Here the subscript *L* stands for Langmuir and means that the parameters of this looking-like-in-the-rest charge carrier enter the plasma frequency (2.4), (2.5). That allows fulfilling the requirement of zero kinetic (true) momentum.

The boundary conditions in directions lateral to the boundary plane are not fixed at any determinate spatial points for the surface impedance (plane wave) approximation, so the electroneutrality requirement should result in the local implementation of this requirement for the moving intrinsic charge. To assure it simultaneously with the above-considered condition of zero kinetic (true) momentum and regardless of lateral boundary conditions let us compare the electric dipole moments in both the first and the second interior coordinate systems, which have been introduced here. As it was mentioned previously, the electric dipole moments acquired by a charge in a time interval t_0 should be the same in both interior coordinate systems due to an absence of extrinsic charges $q_m \Delta x_m = q_L \Delta x_L$. Here $\Delta x_m = v_m t_0$, $\Delta x_L = v_L t_0$, v_m and v_L are the respective velocities of the charge movement in the first interior coordinate system, relative to which a charge is moving, and in the second interior coordinate system, relative to which the charge looks like immobile in the exterior rest reference coordinates. The electric charge of the immobile carrier q_L corresponds to the charge value entering the Langmuir plasma frequency (2.4), (2.5).

The condition of local neutrality for the moving intrinsic charge, which acts as a discussed above integrator of the incoming specific linear density η, may be formulated as a predicate of the local even parity of a spatial distribution of the bare linear density ρ of the background charge of the opposite sign in a vicinity of the location point of moving charge. Such a local symmetry of the background ρ regions around the peak η acquired by the charge q_m moving over these symmetric sections of the bare, noncompensated background allows remaining the global electroneutrality irrespectively of lateral boundary conditions. The desired symmetry of the bare background sections can be assured by a movement of the mobile charge with the velocity v_m relative to the first coordinate system, which itself should move in the opposite direction with the value of velocity $v_L = 2v_m$ relative to the exterior rest coordinates. A moving intrinsic charge q_m would integrate the incoming specific linear density of mobile charges η stripping it off both sides around this moving charge. The section of bare background on the one side would be formed due to the charge movement together with the first coordinate system with the velocity value $v_L = 2v_m$. The same section on the opposite side would be formed due to the movement of the charge itself relative to the first coordinate system with the velocity value v_m. Substituting this ratio of velocities in the equality of dipole moments in both intrinsic coordinate systems $q_m \Delta x_m = q_L \Delta x_L$ in the form of $\Delta x_m = v_m t_0$, $\Delta x_L = v_L t_0$ one obtains the relations $q_m \cdot v_m t_0 = q_L \cdot v_L t_0 = q_L \cdot 2v_m t_0$, i. e. the double ratio $q_m = 2q_L$ for the charges in the first and second intrinsic coordinate systems. Since the charge q_L enters the Langmuir plasma frequency expressions (2.4), (2.5), the charge $q_m = 2q_L$ being substituted in these expressions gives the double ratio for penetration depths and wave numbers $k''_{ef} = 2k''_A$ in the force expressions (3.4) and (3.8). So it may be argued that at least two coordinate systems should be introduced in an electrodynamics problem to describe the nonlocality properly. It would be analogous with the situation of two travelling waves, which could compose one standing wave, and vice versa.

The double ratio $k''_{ef} = 2k''_A$ in the force expressions (3.4) and (3.8) results from the non-dissipating direct current, which provides the Meissner effect and may be presented as the superposition of plasmon polaritons with positive and negative frequencies, corresponding to the zero total frequency, and with equal wavenumbers k''_0, corresponding to the double total wavenumber $2k''_0$ (cf. Eqs (2.6) and (2.7)). The comprehensible generally accepted scenario emerges from the given explanations, according to which the Meissner effect in superconductors is the result of an incomplete cancellation of the diamagnetic and the paramagnetic currents in response to an external magnetic induction field $\mathbf{B}$ at temperatures below the transition temperature.

Such "pairing" effect should be inherent both for superconductors and for "normal" perfect conductors. This conclusion is supported by the experimental data [17] for mesoscopic conductors and probably [18] for nonequilibrium electron-hole droplets.

6 LONDON EQUATIONS AND ABRAHAM FORCE

The partial derivative of magnetic induction with respect to time in the second of Maxwell equations (3.6) may be represented as

$$(\partial\widetilde{\mathbf{B}}/\partial t) = -\mathrm{rot}\ \widetilde{\mathbf{E}} = -\mathrm{rot}\ (\widetilde{\mathbf{j}}/\widetilde{\sigma}) = -(i\omega\mu_0/(k_0'')^2)\mathrm{rot}\,\widetilde{\mathbf{j}}, \tag{6.1}$$

where the first London equation [2, 10, 11] $d\widetilde{\mathbf{j}}\,(z,t)/dt = d(\widetilde{\mathbf{j}}\,(z)e^{i\omega t})/dt = i\omega\widetilde{\mathbf{j}}\,(z,t) = (1/\mu_0\lambda_0^2)\widetilde{\mathbf{E}} = ((k_0'')^2/\mu_0)\widetilde{\mathbf{E}}$ has been used to derive the conductivity $\widetilde{\sigma}$ from Ohm's law $\widetilde{\mathbf{j}} = \widetilde{\sigma}\widetilde{\mathbf{E}} = (1/i\omega\mu_0\lambda_0^2)\widetilde{\mathbf{E}} = ((k_0'')^2/i\omega\mu_0)\widetilde{\mathbf{E}}$. Substituting the Eq. (6.1) and second London equation [2, 11] $\mathrm{rot}\ \widetilde{\mathbf{j}} = -(1/\lambda_0^2)\widetilde{\mathbf{H}} = -(k_0'')^2\widetilde{\mathbf{H}}$ into the last term of Abraham force (3.5), one gets the reinforced expression instead of the Eq. (3.8)

$$\begin{aligned}
\left(1-\frac{1}{\widetilde{\widetilde{\varepsilon}}}\right)\left[\frac{\partial\widetilde{\mathbf{D}}}{\partial t}\times\widetilde{\mathbf{B}}\right] + \left(1-\frac{1}{\widetilde{\widetilde{\varepsilon}}}\right)\left[\widetilde{\mathbf{D}}\times\frac{\partial\widetilde{\mathbf{B}}}{\partial t}\right] &= \\
= \left(1-\frac{1}{\widetilde{\widetilde{\varepsilon}}}\right)\left[\mathrm{rot}\,\widetilde{\mathbf{H}}\times\widetilde{\mathbf{B}}\right] - \left(1-\frac{1}{\widetilde{\widetilde{\varepsilon}}}\right)\left[\widetilde{\mathbf{D}}\times\mathrm{rot}\,\widetilde{\mathbf{E}}\right] &= \\
= \left(1-\frac{1}{\widetilde{\widetilde{\varepsilon}}}\right)\left[\mathrm{rot}\,\widetilde{\mathbf{H}}\times\widetilde{\mathbf{B}}\right] - \left(1-\frac{1}{\widetilde{\widetilde{\varepsilon}}}\right)\left[\widetilde{\mathbf{D}}\times\frac{i\omega\mu_0}{(k_0'')^2}\,\mathrm{rot}\,\widetilde{\mathbf{j}}\right] &= \\
= \left(1-\frac{1}{\widetilde{\widetilde{\varepsilon}}}\right)\left[\mathrm{rot}\,\widetilde{\mathbf{H}}\times\widetilde{\mathbf{B}}\right] - \left(1-\frac{1}{\widetilde{\widetilde{\varepsilon}}}\right)\left[\widetilde{Z}\,\widetilde{\mathbf{D}}\times\frac{1}{k_0''}\,\mathrm{rot}\,\widetilde{\mathbf{j}}\right] &= \\
= \left(1-\frac{1}{\widetilde{\widetilde{\varepsilon}}}\right)\left[\mathrm{rot}\,\widetilde{\mathbf{H}}\times\widetilde{\mathbf{B}}\right] + \left(1-\frac{1}{\widetilde{\widetilde{\varepsilon}}}\right)\left[\widetilde{B}_y(0)\mathbf{i}\times k_0''\,\widetilde{\mathbf{H}}\right] &= \\
= \left(1-\frac{1}{\widetilde{\widetilde{\varepsilon}}}\right)\mu_0\left[\mathrm{rot}\,\widetilde{\mathbf{H}}\times\widetilde{\mathbf{H}}\right] + \left(1-\frac{1}{\widetilde{\widetilde{\varepsilon}}}\right)\left[\frac{k_0''}{\mu_0}\widetilde{B}_y(0)\mathbf{i}\times\widetilde{B}_y(0)\mathbf{j}\right] &= \\
= 2\left(1-\frac{1}{\widetilde{\widetilde{\varepsilon}}}\right)\left[\frac{k_0''}{\mu_0}\widetilde{B}_y(0)\mathbf{i}\times\widetilde{B}_y(0)\mathbf{j}\right]&.
\end{aligned} \tag{6.2}$$

Here the modulus (Eq. (2.9)) and argument $\psi_1 = \pi/2$ of the complex surface impedance $\widetilde{Z}$ have been used similarly to the Eq. (3.4). Comparisons of the Eqs (3.4), (3.8) and (6.2) reveal the double ratios $k_{ef}'' = 2k_A'' = 2k_0''$ in the force expressions.

The equal values of penetration depths and wave numbers $k_0'' = k_A''$ in the force expressions (3.4) and (6.2) show that the Abraham force in the Eq. (3.4) in the stationary limit $\omega \to 0$ correctly deals with the role of the gauge degrees of freedom to define the fields $\mathbf{H}$ and $\mathbf{D}$. The fixed argument and modulus of a complex surface impedance $\widetilde{Z}$ determined by conditions of the spatial force resonances are obviously significant in gauge fixing. Such "gauge fixing" in the Eq. (3.4) to the London gauge [11], postulated in the force expression (6.2) by London equations, can be clarified by formal backgrounds of the electrodynamic nonlocality described in the Sec. 4. Backgrounds of the spatial nonlocality are closely relevant to the principles of gauge theories such as the configuration space, redundant degrees of freedom, classes. However, the Sec.4 is addressed mainly to analyse the nonself-adjoint problems and further detailed formal comparisons of approaches presented here, and in gauge theories are beyond the scope of this study.

The last notice also concerns the following conclusion. The equalities (3.4), (3.8) and (6.2) demonstrate that finding the proper, empirically verified solution of the electrodynamic problem may require an introduction of the effective constitutive parameters (cf. Sec. 5) or postulation of the additional phenomenological equations (cf. Sec. 6) depending on the kind of representation of the original problem operator. The reasons for the dependence on operator representation can be found also in the Sec. 4. It has been argued that the combined vector manifold of the real three-dimensional linear space and the empiric physical fields transforms into the real nonlinear three-dimensional vector manifold. This can affect the linear properties and commutativity of operators acting in this nonlinear manifold. In turn, the discussed dependence on operator representation can be the result of lost linearity and commutativity especially in the case of evaluating the indeterminate form in the zero frequency limit. It will be considered in details elsewhere.

7 CONCLUSIONS

The study derived the general frequency dependence of the frequency-dependent surface impedance for the solutions corresponding to the spatially dispersive eigenvalues of the permittivity operator for conductors for all solutions including that for superconductors. It is shown that an incorporation of the spatial dispersion leads to an appearance of the Meissner effect in perfect conductors in the same manner as in superconductors. Formal backgrounds of the electrodynamic nonlocality were deduced. This expanded conception is promising for applications in novel nanoelectronic devices exploiting the coherence, nonlocality of the superconducting-like state and for search of approaches to the problem of room temperature superconductivity. The obtained results demonstrate that finding the proper, empirically verified solution of the electrodynamic problem may require an introduction of the effective constitutive parameters or postulating the additional phenomenological equations in accordance with the kind of formal representation of the original problem.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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