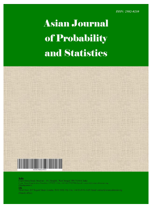


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# A Study on Second Order Rotatable Designs under Tri-diagonal Correlated Structure of Errors Using a Pair of Balanced Incomplete Block Designs

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***Authors' contributions***

*This work was carried out in collaboration between both authors. Author KRS designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author BRV managed the analyses of the study and the literature searches. Both authors read and approved the final manuscript.*

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## Abstract

In this paper, a study on second order rotatable designs under tri-diagonal correlated structure of errors using a pair of balanced incomplete block designs is suggested. Further, the variance function of the estimated response for different values of tri-diagonal correlated coefficient $(\rho)$ and distance from centre $(d)$ for $5 \le v \le 15$ ($v$ - factors) are studied.

*Keywords: Second order rotatable designs; tri-diagonal correlated errors; balanced incomplete block designs.*

## 1 Introduction

Response surface methodology (RSM) often deals with a natural and desirable property rotatability, which requires that, the variance of the predicted response at a point remains constant at all such points that are

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equidistant from the design center. To achieve stability in prediction variance, this important property of rotatability was developed.

The concept of rotatability, which is very important in response surface designs was, introduced by Box and Hunter [1]. Das and Narasimham [2] constructed, second order rotatable designs (SORD) through balanced incomplete block designs (BIBD). Narasimham, et al. [3] constructed SORD using a pair of BIBD. Panda and Das [4] introduced first order rotatable designs with correlated errors. Das [5,6,7] introduced and studied Robust Second Order Rotatable Designs (RSORD).

Rajyalakshmi [8] studied some contributions to second order rotatable and slope rotatable response surface designs under different correlated error structures. Rajyalakshmi and Victorbabu [9,10,11] studied SORD under tri-diagonal correlated structure of errors using central composite designs, incomplete block designs and BIBD respectively. Rajyalakshmi, Prasanthi and Victorbabu [12] suggested an empirical study of SORD under tri-diagonal correlated structure of errors using a pair of partially balanced incomplete block type designs. Sulochana and Victorbabu [13] studied Second order slope rotatable designs under tri-diagonal correlation structure of errors using a pair of incomplete block designs.

Ravindrababu and Victorbabu [14] studied SORD under intra class correlated structure of errors using a pair of balanced incomplete block designs. Ravindrababu and Victorbabu [15] studied SORD under intra class correlated structure of errors using a pair of symmetrical unequal block arrangements with two unequal block sizes. Rajyalakshmi and Victorbabu [16] constructed SOSRD under intra-class correlated error structure using balanced incomplete block designs. Sulochana and Victorbabu [17] studied SOSRD under intra-class correlated structure of errors using a pair of balanced incomplete block designs. Sulochana and Victorbabu [18] studied SOSRD under intra- class correlated structure of errors using partially balanced incomplete block type designs.

In this paper, a study on second order rotatable designs under tri-diagonal correlated structure of errors using a pair of balanced incomplete block designs is suggested and the variance of the estimated response for different values of the tri-diagonal correlated coefficient $(\rho)$ and the distance from the centre $(d)$ for V factors $5 \leq v \leq 15$ are studied.

## 2 Conditions for Second Order Rotatable Designs under Tri-diagonal Correlated Structure of Errors

A second order response surface design $D=((x_{iu}))$ for fitting

$$Y_u(x)=b_0+\sum_{i=1}^{v} b_i x_{iu}+\sum_{i=1}^{v} b_{ii} x_{iu}^2+\sum_{i=1}^{v}\sum_{i<j=1}^{v} b_{ij} x_{iu} x_{ju}+e_u \tag{1}$$

where $x_{iu}$ denotes the level of the $i^{th}$ factor $(i=1,2,....,v)$ in the $u^{th}$ run $(u=1,2,....,N)$ of the experiment, $e_u$ 's are correlated random errors is said to be a SORD under tri-diagonal correlated structure of errors, if the variance of the estimated response of $\hat{Y}_u$ from the fitted surface is only a function of the distance $\left(d^2=\sum_{i=1}^{v} x_i^2\right)$ of the point $(x_{1u}, x_{2u}, \ldots, x_{vu})$ from the origin (center) of the design. Such a spherical variance function for estimation of responses in the second order response surface is achieved if the design points satisfy the following conditions [Das [7,19], Rajyalakshmi, and Victorbabu [9,10,11].

$$\sum_{u=1}^{2n} \prod_{i=1}^{v} x_{iu}^{\alpha_i} = 0 \text{ if any } \alpha_i \text{ is odd and } \sum \alpha_i \le 4 \quad (2)$$

$$\sum_{u=1}^{2n} x_{iu}^2 = \text{constant} = 2n\gamma_2 \quad (3)$$

$$\sum_{u=1}^{2n} x_{iu}^4 = \text{constant} = 3(2n)\gamma_4 \quad (4)$$

$$\sum_{u=1}^{2n} x_{iu}^2 x_{ju}^2 = \text{constant} = 2n\gamma_4 \quad \text{for all } i \ne j \quad (5)$$

$$\sum_{u=1}^{2n} x_{iu}^4 = 3\sum_{u=1}^{2n} x_{iu}^2 x_{ju}^2 \quad (6)$$

$$\frac{\gamma_4}{\gamma_2^2} > \frac{v(1-\rho)}{(v+2)} \text{ ( Non – singularity condition)} \quad (7)$$

where $\gamma_2$ and $\gamma_4$ are constants and the summation is over the design points.

The Variances and Covariances of the estimated parameters are as follows:

$$V(\hat{b}_0) = \frac{\gamma_4(v+2)(1+\rho)\sigma^2}{2n[\gamma_4(v+2)-v\gamma_2^2(1-\rho)]}$$

$$V(\hat{b}_i) = \frac{\sigma^2(1-\rho^2)}{2n\gamma_2}$$

$$V(\hat{b}_{ij}) = \frac{\sigma^2(1-\rho^2)}{2n\gamma_4}$$

$$V(\hat{b}_{ii}) = \frac{\sigma^2(1-\rho^2)[\gamma_4(v+1)-(v-1)\gamma_2^2(1-\rho)]}{2(2n)\gamma_4[\gamma_4(v+2)-v\gamma_2^2(1-\rho)]}$$

$$cov(\hat{b}_0, \hat{b}_{ii}) = \frac{-\gamma_2\sigma^2(1-\rho^2)}{2n[\gamma_4(v+2)-v\gamma_2^2(1-\rho)]}$$

$$cov(\hat{b}_{ii}, \hat{b}_{jj}) = \frac{\sigma^2(1-\rho^2)[\gamma_2^2(1-\rho)-\gamma_4]}{2(2n)\gamma_4[\gamma_4(v+2)-v\gamma_2^2(1-\rho)]} \quad (8)$$

and other covariances are zero.

The variance of the estimated response $\hat{Y}_u$ at any point estimated through the surface comes out as,

$$V(\hat{Y}_u)=V(\hat{b}_0)+d^2[V(\hat{b}_i)+2\,cov(\hat{b}_0,\hat{b}_{ii})]+d^4\,V(\hat{b}_{ii}) \tag{9}$$

Hence the variance of estimate of $\hat{Y}_u$ becomes,

$$V(\hat{Y}_u)=\frac{[\gamma_4(v+2)(1+\rho)\sigma^2]}{2n[\gamma_4(v+2)-v\gamma_2^2(1-\rho)]}+\left[\frac{\sigma^2(1-\rho^2)}{2n\gamma_2}+2\left(-\frac{\gamma_2\sigma^2(1-\rho^2)}{2n[\gamma_4(v+2)-v\gamma_2^2(1-\rho)}\right)\right]d^2+\frac{\sigma^2(1-\rho)[\gamma_4(v+1)-(v-1)\gamma_2^2(1-\rho)]}{2(2n)\gamma_4[\gamma_4(v+2)-v\gamma_2^2(1-\rho)]}d^4 \tag{10}$$

## 3 Some Preliminaries

**(i) Tri-diagonal structure**:

It is a covariance structure of errors which is a relaxation of intra-class structure or log model covariance structure of errors and is given by(cf. Das [7])

$$w_0=\left\{D(e)=\sigma^2\left[\begin{pmatrix}I_n & I_n\\ I_n & I_n\end{pmatrix}\times\frac{1+\rho}{2}+\begin{pmatrix}I_n & -I_n\\ -I_n & I_n\end{pmatrix}\times\frac{1-\rho}{2}\right]=w_{2n\times 2n}(\rho)\right\}, \tag{i}$$

$$w_{2n\times 2n}^{-1}(\rho)=(\sigma^2)^{-1}\left[\begin{pmatrix}I_n & I_n\\ I_n & I_n\end{pmatrix}\times\frac{1}{2(1+\rho)}+\begin{pmatrix}I_n & -I_n\\ -I_n & I_n\end{pmatrix}\times\frac{1}{2(1-\rho)}\right]. \tag{ii}$$

**(ii) Second order rotatable design**

A second order response surface design as given in equation (1) where $x_{iu}$ denotes the level of the $i^{th}$ factor ($i=1,2,....,v$) in the $u^{th}$ run ($u=1,2,....,N$) of the experiment, $e_u$'s are uncorrelated random errors with mean zero and variance $\sigma^2$. The design D is said to be a SORD, if the variance of the estimated response of $\hat{Y}_u$ from the fitted surface is only a function of the distance $(d^2=\sum_{i=1}^{v} x_i^2)$ of the point $(x_{1u},x_{2u},....,x_{vu})$ from the origin (centre) of the design ( cf. Box and Hunter [1], Das and Narasimham [2]).

## 4 Method of Construction of SORD under Tri-diagonal Correlated Structure of Errors Using a Pair of BIBD

The method of construction of SORD using two suitably chosen BIBD, can be obtained as follows (cf. Narasimham et al. [3])

**Result:** Let $D_1=(v,b_1,r_1,k_1,\lambda_1)$ and $D_2=(v,b_2,r_2,k_2,\lambda_2)$ are two balanced incomplete block designs in 'v' factors with $r_1 \underset{>}{\overset{<}{-}} 3\lambda_1$ and $r_2 \underset{<}{\overset{>}{-}} 3\lambda_2$ respectively, then the design points, $[1\text{-}(v,b_1,r_1,k_1,\lambda_1)]\times 2^{t(k_1)} U [a\text{-}(v,b_2,r_2,k_2,\lambda_2)]\times 2^{t(k_2)} U n_0$ give a v-dimensional SORD in $N=b_1\times 2^{t(k_1)}+b_2\times 2^{t(k_2)}$ design points with $a^4=-\frac{(r_1-3\lambda_1)}{(r_2-3\lambda_2)}\times 2^{t(k_1)-t(k_2)}$.

Following Das and Narasimham [2], Narasimham et al. [3] and Das [5,6,7] methods of constructions of SORD and Robust second order rotatable design, here we study a new method of construction of SORD under tri-diagonal correlated error structure using a pair of BIBD.

Let $D_1=(v,b_1,r_1,k_1,\lambda_1)$ and $D_2=(v,b_2,r_2,k_2,\lambda_2)$ denote a pair of BIBD, $2^{t(k_1)}$ denote a fractional replicate of $2^{k_1}$ in +1 and -1 levels, in which no interaction with less than five factors is confounded. Let $[1-(v,b_1,r_1,k_1,\lambda_1)]$ denote the design points generated from the transpose of incidence matrix of a BIBD. $[1-(v,b_1,r_1,k_1,\lambda_1)]\times 2^{t(k_1)}$ are the $b_1\times 2^{t(k_1)}$ design points generated from a BIBD by "multiplication" in Das and Narasimham [2] sense. Let $[a-(v,b_2,r_2,k_2,\lambda_2)]\times 2^{t(k_2)}$ are the $b_2\times 2^{t(k_2)}$ design points and $\bigcup$ denotes combination of the design points generated from different sets of points. $n_0$ denote the number of central points.

The proposed method of construction of SORD under tri-diagonal correlated structure of errors using a pair of BIBD is given below. Considering the method of construction of SORD using a pair of BIBD having $b_1\times 2^{t(k_1)}+b_2\times 2^{t(k_2)}$ (cf. Narasimham et al. [3]) non-central design points $(n)$ .The set of 'n' non central design points are extended to 2n design points (N) by adding 'n' ($n_0$=n) central points $(0,0,....,0)$ just below or above the 'n' non-central design points. Hence, $2n(=N)$ be the total number of design points of the SORD under tri-diagonal correlated structure of errors using a pair of BIBD.

**Theorem:** The design points, $[1\text{-}(v,b_1,r_1,k_1,\lambda_1)\times 2^{t(k_1)} U [a\text{-}(v,b_2,r_2,k_2,\lambda_2)]\times 2^{t(k_2)} U n_0$ will give a v-dimensional SORD under tri-diagonal correlated structure of errors using a pair of BIBD in $N=b_1\times 2^{t(k_1)}+b_2\times 2^{t(k_2)}+n_0$ design points, with $a^4=-\frac{(r_1-3\lambda_1)}{(r_2-3\lambda_2)}\times 2^{t(k_1)-t(k_2)}$.

**Proof:** For the design points generated from a pair of BIBD, the simple symmetry conditions (2) are true. Further, the conditions (3), (4) and (5) are true as follows:

$$\sum_{u=1}^{2n} x_{iu}^2 = r_1 2^{t(k_1)} + r_2 2^{t(k_2)} a^2 = \text{constant} = 2n\gamma_2 \tag{11}$$

$$\sum_{u=1}^{2n} x_{iu}^4 = r_1 2^{t(k_1)} + r_2 2^{t(k_2)} a^4 = \text{constant} = 3(2n)\gamma_4 \tag{12}$$

$$\sum_{u=1}^{2n} x_{iu}^2 x_{ju}^2 = \lambda_1 2^{t(k_1)} + \lambda_2 2^{t(k_2)} a^4 = \text{constant} = 2n\gamma_4 \tag{13}$$

From (12) and (13) we have,

$$r_1 2^{t(k_1)} + r_2 2^{t(k_2)} a^4 = 3\left(\lambda_1 2^{t(k_1)} + \lambda_2 2^{t(k_2)} a^4\right) \tag{14}$$

$$\text{(i.e.,)}\quad (r_1 - 3\lambda_1)2^{t(k_1)} + (r_2 - 3\lambda_2)2^{t(k_2)} a^4 = 0 \tag{15}$$

If $r_1 = 3\lambda_1$ and $r_2 = 3\lambda_2$, above equation (15) is satisfied. But to get a non- trivial design, at least one of levels '1' or 'a' must be different from 0 (zero).

If $r_1 \neq 3\lambda_1$ and $r_2 \neq 3\lambda_2$, above equation(15) gives ,

$$a^4 = -\frac{(r_1 - 3\lambda_1)}{(r_2 - 3\lambda_2)} \times 2^{t(k_1) - t(k_2)}. \tag{16}$$

Equation (16) has a real solution if either $r_1 < 3\lambda_1$ or $r_2 < 3\lambda_2$, but not both.

If the non-singularity condition (7) exists then only the design exists.

**Example :** A study on second order rotatable designs under tri-diagonal correlated structure of errors using a pair of balanced incomplete block designs can explained using v=5 factor with $D_1 = (v=5, b_1=5, r_1=4, k_1=4, \lambda_1=3)$ and $D_2 = (v=5, b_2=10, r_2=4, k_2=2, \lambda_2=1)$.

The design points, $[1-(5,5,4,4,3)] \times 2^4 U[(a-(5,10,4,2,1)] \times 2^2 U(n_0=120)$ will give a v-dimensional SORD under tri-diagonal correlated structure of errors using a pair of BIBD in $N = 240$ design points, from (3), (4), and (5) we have,

$$\sum_{u=1}^{2n} x_{iu}^2 = 64 + 16a^2 = 2n\gamma_2 \tag{17}$$

$$\sum_{u=1}^{2n} x_{iu}^4 = 64 + 16a^4 = 3(2n)\gamma_4 \tag{18}$$

$$\sum_{u=1}^{2n} x_{iu}^2 x_{ju}^2 = 48+4a^4 = 2n\gamma_4 \quad (19)$$

On simplification of (18) and (19), we get a=2.114742. By substituting value of ‘a’ in (17) and (18) we obtain $\gamma_2=0.564809$ and $\gamma_4=0.533333$ . Non-singularity condition (7) is also satisfied as 1.671843>0.642857.

From (8), (9) and (10) we have,

$$V(\hat{Y}_u)=V(\hat{b}_0)+d^2[V(\hat{b}_i)+2\,cov(\hat{b}_0,\hat{b}_{ii})]+d^4\,V(\hat{b}_{ii})$$

$$V(\hat{Y}_u)=\frac{3.733331(1+\rho)\sigma^2}{240[3.733331-1.595046(1-\rho)]}+[\frac{\sigma^2(1-\rho^2)}{135.555416}+2(\frac{-0.564809\sigma^2(1-\rho^2)}{240[3.733331-1.595046(1-\rho)]}]d^2$$

$$+\frac{\sigma^2(1-\rho^2)[3.199998-1.276036(1-\rho)]}{255.9998[3.733331-1.595046(1-\rho)]}d^4. \quad (20)$$

The variance function of SORD under tri-diagonal correlated structure of errors using a pair of BIBD at different design points for factors $5 \le v \le 15$ are given appendix in Table 1.

## 5 A Study of Dependence of the Variance Function of the Response at Different Design Points

Here, we study the dependence of variance function of response at different design points for SORD under tri-diagonal correlated structure of errors using a pair of BIBD. Given ‘v’ factors different values of tri-diagonal correlated coefficient $\rho$ and distance from center ‘d’ (centre of the design) between 0 and 1, the variances are tabulated.

From (20) the variance of the estimated response is obtained by

$$V(\hat{Y}_u)=0.00749985 \text{ (by taking } \rho=0.1,\ d=0.1 \text{ and } \sigma=1)$$

For a given $v$, the study of variance function of response at different design points for SORD under tri-diagonal correlated structure of errors using pair of balanced incomplete block designs for $5 \le v \le 15$ and distance from center $d$ for $d=0.1(0.1)1.0$ are tabulated.

The numerical calculations are given in Table 2.

The graphical representation for SORD under tri-diagonal correlated structure of errors using a pair of BIBD for 5-factors in given in the appendix.

**Table 1. The variance function of SORD under tri-diagonal correlated structure of errors using a pair of BIBD at different design points for factors $5 \leq v \leq 15$**

| $\rho$ | $D_1$=(v=5,$b_1$=5,$r_1$=4,$k_1$=4,$\lambda_1$=3) $D_2$=(v=5,$b_2$=10,$r_2$=4,$k_2$=2,$\lambda_2$=1), N=240 | $D_1$=(v=6,$b_1$=6,$r_1$=5,$k_1$=5,$\lambda_1$=4) $D_2$=(v=6,$b_2$=15,$r_2$=5,$k_2$=2,$\lambda_2$=1),N=312 | $D_1$=(v=7,$b_1$=7,$r_1$=4,$k_1$=4,$\lambda_1$=2) $D_2$=(v=7,$b_2$=21,$r_2$=6,$k_2$=2,$\lambda_2$=1),N=392 |
|---|---|---|---|
| -0.9 | $0.0022\sigma^2+0.0001\sigma^2d^2+0.0008\sigma^2d^4$ | $0.0036\sigma^2-0.0010\sigma^2d^2+0.0010\sigma^2d^4$ | $0.0043\sigma^2-0.0025\sigma^2d^2+0.0031\sigma^2d^4$ |
| -0.8 | $0.0036\sigma^2+0.0006\sigma^2d^2+0.0014\sigma^2d^4$ | $0.0047\sigma^2-0.0004\sigma^2d^2+0.0017\sigma^2d^4$ | $0.0046\sigma^2-0.0010\sigma^2d^2+0.0048\sigma^2d^4$ |
| -0.7 | $0.0045\sigma^2+0.0014\sigma^2d^2+0.0020\sigma^2d^4$ | $0.0052\sigma^2+0.0004\sigma^2d^2+0.0022\sigma^2d^4$ | $0.0048\sigma^2+0.0005\sigma^2d^2+0.0063\sigma^2d^4$ |
| -0.6 | $0.0052\sigma^2+0.0021\sigma^2d^2+0.0024\sigma^2d^4$ | $0.0055\sigma^2+0.0012\sigma^2d^2+0.0027\sigma^2d^4$ | $0.0049\sigma^2+0.0019\sigma^2d^2+0.0075\sigma^2d^4$ |
| -0.5 | $0.0058\sigma^2+0.0028\sigma^2d^2+0.0028\sigma^2d^4$ | $0.0057\sigma^2+0.0020\sigma^2d^2+0.0030\sigma^2d^4$ | $0.0049\sigma^2+0.0032\sigma^2d^2+0.0086\sigma^2d^4$ |
| -0.4 | $0.0062\sigma^2+0.0035\sigma^2d^2+0.0030\sigma^2d^4$ | $0.0058\sigma^2+0.0027\sigma^2d^2+0.0033\sigma^2d^4$ | $0.0049\sigma^2+0.0043\sigma^2d^2+0.0094\sigma^2d^4$ |
| -0.3 | $0.0065\sigma^2+0.0041\sigma^2d^2+0.0033\sigma^2d^4$ | $0.0059\sigma^2+0.0033\sigma^2d^2+0.0035\sigma^2d^4$ | $0.0050\sigma^2+0.0053\sigma^2d^2+0.0100\sigma^2d^4$ |
| -0.2 | $0.0068\sigma^2+0.0045\sigma^2d^2+0.0034\sigma^2d^4$ | $0.0060\sigma^2+0.0038\sigma^2d^2+0.0037\sigma^2d^4$ | $0.0050\sigma^2+0.0060\sigma^2d^2+0.0105\sigma^2d^4$ |
| -0.1 | $0.0070\sigma^2+0.0049\sigma^2d^2+0.0035\sigma^2d^4$ | $0.0061\sigma^2+0.0042\sigma^2d^2+0.0038\sigma^2d^4$ | $0.0050\sigma^2+0.0066\sigma^2d^2+0.0107\sigma^2d^4$ |
| 0 | $0.0072\sigma^2+0.0051\sigma^2d^2+0.0035\sigma^2d^4$ | $0.0061\sigma^2+0.0044\sigma^2d^2+0.0038\sigma^2d^4$ | $0.0050\sigma^2+0.0069\sigma^2d^2+0.0107\sigma^2d^4$ |
| 0.1 | $0.0074\sigma^2+0.0052\sigma^2d^2+0.0034\sigma^2d^4$ | $0.0062\sigma^2+0.0045\sigma^2d^2+0.0037\sigma^2d^4$ | $0.0050\sigma^2+0.0071\sigma^2d^2+0.0106\sigma^2d^4$ |
| 0.2 | $0.0075\sigma^2+0.0052\sigma^2d^2+0.0033\sigma^2d^4$ | $0.0062\sigma^2+0.0045\sigma^2d^2+0.0036\sigma^2d^4$ | $0.0050\sigma^2+0.0071\sigma^2d^2+0.0102\sigma^2d^4$ |
| 0.3 | $0.0077\sigma^2+0.0050\sigma^2d^2+0.0031\sigma^2d^4$ | $0.0062\sigma^2+0.0044\sigma^2d^2+0.0033\sigma^2d^4$ | $0.0050\sigma^2+0.0069\sigma^2d^2+0.0096\sigma^2d^4$ |
| 0.4 | $0.0078\sigma^2+0.0047\sigma^2d^2+0.0028\sigma^2d^4$ | $0.0063\sigma^2+0.0042\sigma^2d^2+0.0031\sigma^2d^4$ | $0.0050\sigma^2+0.0065\sigma^2d^2+0.0088\sigma^2d^4$ |
| 0.5 | $0.0079\sigma^2+0.0043\sigma^2d^2+0.0025\sigma^2d^4$ | $0.0063\sigma^2+0.0038\sigma^2d^2+0.0027\sigma^2d^4$ | $0.0050\sigma^2+0.0059\sigma^2d^2+0.0079\sigma^2d^4$ |
| 0.6 | $0.0080\sigma^2+0.0037\sigma^2d^2+0.0021\sigma^2d^4$ | $0.0063\sigma^2+0.0033\sigma^2d^2+0.0023\sigma^2d^4$ | $0.0050\sigma^2+0.0051\sigma^2d^2+0.0067\sigma^2d^4$ |
| 0.7 | $0.0081\sigma^2+0.0030\sigma^2d^2+0.0017\sigma^2d^4$ | $0.0063\sigma^2+0.0026\sigma^2d^2+0.0018\sigma^2d^4$ | $0.0050\sigma^2+0.0041\sigma^2d^2+0.0053\sigma^2d^4$ |
| 0.8 | $0.0082\sigma^2+0.0021\sigma^2d^2+0.0012\sigma^2d^4$ | $0.0063\sigma^2+0.0019\sigma^2d^2+0.0013\sigma^2d^4$ | $0.0050\sigma^2+0.0029\sigma^2d^2+0.0037\sigma^2d^4$ |
| 0.9 | $0.0082\sigma^2+0.0011\sigma^2d^2+0.0006\sigma^2d^4$ | $0.0063\sigma^2+0.0010\sigma^2d^2+0.0006\sigma^2d^4$ | $0.0050\sigma^2+0.0015\sigma^2d^2+0.0019\sigma^2d^4$ |

| $\rho$ | $D_1$=(v=8,$b_1$=14,$r_1$=7,$k_1$=4,$\lambda_1$=3) $D_2$=(v=8,$b_2$=28,$r_2$=7,$k_2$=2,$\lambda_2$=1), N=672 | $D_1$=(v=10,$b_1$=18,$r_1$=9,$k_1$=45,$\lambda_1$=4) $D_2$=(v=10,$b_2$=45,$r_2$=9,$k_2$=2,$\lambda_2$=1),N=936 | $D_1$=(v=12,$b_1$=22,$r_1$=11,$k_1$=6,$\lambda_1$=5) $D_2$=(v=12,$b_2$=44,$r_2$=11,$k_2$=3,$\lambda_2$=2), N=2112 |
|---|---|---|---|
| -0.9 | $0.0020\sigma^2-0.0008\sigma^2d^2+0.0020\sigma^2d^4$ | $0.0009\sigma^2+0.0001\sigma^2d^2+0.0013\sigma^2d^4$ | $0.0008\sigma^2-0.0002\sigma^2d^2+0.0005\sigma^2d^4$ |
| -0.8 | $0.0024\sigma^2-0.0008\sigma^2d^2+0.0034\sigma^2d^4$ | $0.0013\sigma^2+0.0007\sigma^2d^2+0.0025\sigma^2d^4$ | $0.0009\sigma^2+0.0001\sigma^2d^2+0.0008\sigma^2d^4$ |
| -0.7 | $0.0026\sigma^2+0.0009\sigma^2d^2+0.0046\sigma^2d^4$ | $0.0015\sigma^2+0.0013\sigma^2d^2+0.0034\sigma^2d^4$ | $0.0009\sigma^2+0.0004\sigma^2d^2+0.0012\sigma^2d^4$ |
| -0.6 | $0.0027\sigma^2+0.0018\sigma^2d^2+0.0056\sigma^2d^4$ | $0.0017\sigma^2+0.0020\sigma^2d^2+0.0042\sigma^2d^4$ | $0.0009\sigma^2+0.0007\sigma^2d^2+0.0014\sigma^2d^4$ |
| -0.5 | $0.0027\sigma^2+0.0026\sigma^2d^2+0.0064\sigma^2d^4$ | $0.0018\sigma^2+0.0026\sigma^2d^2+0.0049\sigma^2d^4$ | $0.0009\sigma^2+0.0009\sigma^2d^2+0.0017\sigma^2d^4$ |
| -0.4 | $0.0028\sigma^2+0.0033\sigma^2d^2+0.0071\sigma^2d^4$ | $0.0018\sigma^2+0.0031\sigma^2d^2+0.0055\sigma^2d^4$ | $0.0009\sigma^2+0.0011\sigma^2d^2+0.0019\sigma^2d^4$ |
| -0.3 | $0.0028\sigma^2+0.0039\sigma^2d^2+0.0076\sigma^2d^4$ | $0.0019\sigma^2+0.0035\sigma^2d^2+0.0059\sigma^2d^4$ | $0.0009\sigma^2+0.0013\sigma^2d^2+0.0020\sigma^2d^4$ |
| -0.2 | $0.0028\sigma^2+0.0044\sigma^2d^2+0.0080\sigma^2d^4$ | $0.0019\sigma^2+0.0038\sigma^2d^2+0.0062\sigma^2d^4$ | $0.0009\sigma^2+0.0014\sigma^2d^2+0.0021\sigma^2d^4$ |
| -0.1 | $0.0028\sigma^2+0.0048\sigma^2d^2+0.0082\sigma^2d^4$ | $0.0019\sigma^2+0.0040\sigma^2d^2+0.0064\sigma^2d^4$ | $0.0009\sigma^2+0.0015\sigma^2d^2+0.0022\sigma^2d^4$ |
| 0 | $0.0029\sigma^2+0.0050\sigma^2d^2+0.0082\sigma^2d^4$ | $0.0020\sigma^2+0.0042\sigma^2d^2+0.0064\sigma^2d^4$ | $0.0009\sigma^2+0.0016\sigma^2d^2+0.0022\sigma^2d^4$ |
| 0.1 | $0.0029\sigma^2+0.0051\sigma^2d^2+0.0081\sigma^2d^4$ | $0.0020\sigma^2+0.0042\sigma^2d^2+0.0063\sigma^2d^4$ | $0.0009\sigma^2+0.0016\sigma^2d^2+0.0021\sigma^2d^4$ |
| 0.2 | $0.0029\sigma^2+0.0050\sigma^2d^2+0.0078\sigma^2d^4$ | $0.0020\sigma^2+0.0041\sigma^2d^2+0.0061\sigma^2d^4$ | $0.0009\sigma^2+0.0016\sigma^2d^2+0.0021\sigma^2d^4$ |
| 0.3 | $0.0029\sigma^2+0.0048\sigma^2d^2+0.0074\sigma^2d^4$ | $0.0020\sigma^2+0.0040\sigma^2d^2+0.0058\sigma^2d^4$ | $0.0009\sigma^2+0.0015\sigma^2d^2+0.0020\sigma^2d^4$ |
| 0.4 | $0.0029\sigma^2+0.0045\sigma^2d^2+0.0068\sigma^2d^4$ | $0.0020\sigma^2+0.0037\sigma^2d^2+0.0053\sigma^2d^4$ | $0.0009\sigma^2+0.0014\sigma^2d^2+0.0018\sigma^2d^4$ |
| 0.5 | $0.0029\sigma^2+0.0041\sigma^2d^2+0.0060\sigma^2d^4$ | $0.0020\sigma^2+0.0033\sigma^2d^2+0.0048\sigma^2d^4$ | $0.0009\sigma^2+0.0013\sigma^2d^2+0.0016\sigma^2d^4$ |
| 0.6 | $0.0029\sigma^2+0.0035\sigma^2d^2+0.0051\sigma^2d^4$ | $0.0021\sigma^2+0.0029\sigma^2d^2+0.0040\sigma^2d^4$ | $0.0009\sigma^2+0.0011\sigma^2d^2+0.0014\sigma^2d^4$ |
| 0.7 | $0.0029\sigma^2+0.0028\sigma^2d^2+0.0041\sigma^2d^4$ | $0.0021\sigma^2+0.0023\sigma^2d^2+0.0032\sigma^2d^4$ | $0.0009\sigma^2+0.0009\sigma^2d^2+0.0011\sigma^2d^4$ |
| 0.8 | $0.0029\sigma^2+0.0020\sigma^2d^2+0.0029\sigma^2d^4$ | $0.0021\sigma^2+0.0016\sigma^2d^2+0.0022\sigma^2d^4$ | $0.0009\sigma^2+0.0006\sigma^2d^2+0.0007\sigma^2d^4$ |
| 0.9 | $0.0029\sigma^2+0.0010\sigma^2d^2+0.0015\sigma^2d^4$ | $0.0021\sigma^2+0.0008\sigma^2d^2+0.0012\sigma^2d^4$ | $0.0009\sigma^2+0.0003\sigma^2d^2+0.0004\sigma^2d^4$ |

| $\rho$ | $D_1$=(v=13,$b_1$=26,$r_1$=12,$k_1$=6,$\lambda_1$=5) $D_2$=(v=13,$b_2$=13,$r_2$=4,$k_2$=4,$\lambda_2$=1), N=2080 | $D_1$=(v=15,$b_1$=15,$r_1$=7,$k_1$=7,$\lambda_1$=3) $D_2$=(v=15,$b_2$=35,$r_2$=7,$k_2$=3,$\lambda_2$=1),N=2480 |
|---|---|---|
| -0.9 | $0.0005\sigma^2+0.0008\sigma^2d^2+0.0003\sigma^2d^4$ | $0.0007\sigma^2-0.0008\sigma^2d^2+0.0004\sigma^2d^4$ |
| -0.8 | $0.0006\sigma^2+0.0003\sigma^2d^2+0.0006\sigma^2d^4$ | $0.0007\sigma^2+0.0002\sigma^2d^2+0.0008\sigma^2d^4$ |
| -0.7 | $0.0007\sigma^2+0.0005\sigma^2d^2+0.0009\sigma^2d^4$ | $0.0007\sigma^2+0.0005\sigma^2d^2+0.0011\sigma^2d^4$ |
| -0.6 | $0.0008\sigma^2+0.0008\sigma^2d^2+0.0012\sigma^2d^4$ | $0.0007\sigma^2+0.0007\sigma^2d^2+0.0013\sigma^2d^4$ |
| -0.5 | $0.0008\sigma^2+0.0013\sigma^2d^2+0.0014\sigma^2d^4$ | $0.0007\sigma^2+0.0009\sigma^2d^2+0.0016\sigma^2d^4$ |
| -0.4 | $0.0008\sigma^2+0.0012\sigma^2d^2+0.0015\sigma^2d^4$ | $0.0007\sigma^2+0.0011\sigma^2d^2+0.0017\sigma^2d^4$ |
| -0.3 | $0.0008\sigma^2+0.0013\sigma^2d^2+0.0016\sigma^2d^4$ | $0.0008\sigma^2+0.0013\sigma^2d^2+0.0019\sigma^2d^4$ |
| -0.2 | $0.0008\sigma^2+0.0014\sigma^2d^2+0.0017\sigma^2d^4$ | $0.0008\sigma^2+0.0014\sigma^2d^2+0.0020\sigma^2d^4$ |
| -0.1 | $0.0009\sigma^2+0.0015\sigma^2d^2+0.0018\sigma^2d^4$ | $0.0008\sigma^2+0.0015\sigma^2d^2+0.0021\sigma^2d^4$ |
| 0 | $0.0009\sigma^2+0.0015\sigma^2d^2+0.0018\sigma^2d^4$ | $0.0008\sigma^2+0.0015\sigma^2d^2+0.0021\sigma^2d^4$ |
| 0.1 | $0.0009\sigma^2+0.0015\sigma^2d^2+0.0018\sigma^2d^4$ | $0.0008\sigma^2+0.0015\sigma^2d^2+0.0020\sigma^2d^4$ |

| $\rho$ | **$D_1$=(v=13,$b_1$=26,$r_1$=12,$k_1$=6,$\lambda_1$=5) $D_2$=(v=13,$b_2$=13,$r_2$=4,$k_2$=4,$\lambda_2$=1), N=2080** | **$D_1$=(v=15,$b_1$=15,$r_1$=7,$k_1$=7,$\lambda_1$=3) $D_2$=(v=15,$b_2$=35,$r_2$=7,$k_2$=3,$\lambda_2$=1),N=2480** |
|---|---|---|
| 0.2 | $0.0009\sigma^2+0.0015\sigma^2d^2+0.0017\sigma^2d^4$ | $0.0008\sigma^2+0.0015\sigma^2d^2+0.0020\sigma^2d^4$ |
| 0.3 | $0.0009\sigma^2+0.0014\sigma^2d^2+0.0016\sigma^2d^4$ | $0.0008\sigma^2+0.0014\sigma^2d^2+0.0019\sigma^2d^4$ |
| 0.4 | $0.0009\sigma^2+0.0013\sigma^2d^2+0.0015\sigma^2d^4$ | $0.0008\sigma^2+0.0013\sigma^2d^2+0.0017\sigma^2d^4$ |
| 0.5 | $0.0009\sigma^2+0.0012\sigma^2d^2+0.0013\sigma^2d^4$ | $0.0008\sigma^2+0.0012\sigma^2d^2+0.0015\sigma^2d^4$ |
| 0.6 | $0.0009\sigma^2+0.0010\sigma^2d^2+0.0011\sigma^2d^4$ | $0.0008\sigma^2+0.0010\sigma^2d^2+0.0013\sigma^2d^4$ |
| 0.7 | $0.0009\sigma^2+0.0008\sigma^2d^2+0.0009\sigma^2d^4$ | $0.0008\sigma^2+0.0008\sigma^2d^2+0.0010\sigma^2d^4$ |
| 0.8 | $0.0009\sigma^2+0.0006\sigma^2d^2+0.0006\sigma^2d^4$ | $0.0008\sigma^2+0.0005\sigma^2d^2+0.0007\sigma^2d^4$ |
| 0.9 | $0.0009\sigma^2+0.0003\sigma^2d^2+0.0003\sigma^2d^4$ | $0.0008\sigma^2+0.0003\sigma^2d^2+0.0003\sigma^2d^4$ |

**Table 2. Study of dependence of estimated SORD under tri-diagonal correlated structure of errors using a pair of BIBD at different design points for $5 \le v \le 15$ for different values of '$\rho$' '$d$' and $\sigma$=1.**

$D_1$=(v=5,$b_1$=5,$r_1$=4,$k_1$=4,$\lambda_1$=3);$D_2$=(v=5,$b_2$=10,$r_2$=4,$k_2$=2,$\lambda_2$=1),N=240

| $\rho$ | d=0.1 | d=0.2 | d=0.3 | d=0.4 | d=0.5 | d=0.6 | d=0.7 | d=0.8 | d=0.9 | d=1 |
|---|---|---|---|---|---|---|---|---|---|---|
| -0.9 | 0.00221 | 0.00221 | 0.00223 | 0.00225 | 0.00229 | 0.00236 | 0.00247 | 0.00263 | 0.00285 | 0.00316 |
| -0.8 | 0.00361 | 0.00363 | 0.00368 | 0.00375 | 0.00387 | 0.00404 | 0.00429 | 0.00465 | 0.00513 | 0.00577 |
| -0.7 | 0.00458 | 0.00462 | 0.00471 | 0.00484 | 0.00504 | 0.00533 | 0.00574 | 0.00629 | 0.00702 | 0.00798 |
| -0.6 | 0.00528 | 0.00535 | 0.00548 | 0.00567 | 0.00596 | 0.00636 | 0.00691 | 0.00765 | 0.00863 | 0.00988 |
| -0.5 | 0.00583 | 0.00592 | 0.00608 | 0.00633 | 0.00670 | 0.00720 | 0.00789 | 0.00880 | 0.00999 | 0.01150 |
| -0.4 | 0.00625 | 0.00636 | 0.00656 | 0.00686 | 0.00730 | 0.00790 | 0.00870 | 0.00976 | 0.01113 | 0.01287 |
| -0.3 | 0.00660 | 0.00673 | 0.00695 | 0.00730 | 0.00779 | 0.00847 | 0.00937 | 0.01055 | 0.01207 | 0.01399 |
| -0.2 | 0.00688 | 0.00702 | 0.00728 | 0.00766 | 0.00820 | 0.00894 | 0.00991 | 0.01119 | 0.01282 | 0.01487 |
| -0.1 | 0.00712 | 0.00727 | 0.00754 | 0.00795 | 0.00852 | 0.00931 | 0.01034 | 0.01167 | 0.01338 | 0.01553 |
| 0 | 0.00732 | 0.00748 | 0.00776 | 0.00819 | 0.00878 | 0.00959 | 0.01065 | 0.01202 | 0.01377 | 0.01596 |
| 0.1 | 0.00749 | 0.00766 | 0.00794 | 0.00837 | 0.00897 | 0.00979 | 0.01085 | 0.01222 | 0.01398 | 0.01617 |
| 0.2 | 0.00764 | 0.00781 | 0.00809 | 0.00851 | 0.00911 | 0.00991 | 0.01096 | 0.01231 | 0.01402 | 0.01616 |
| 0.3 | 0.00777 | 0.00793 | 0.00820 | 0.00861 | 0.00919 | 0.00996 | 0.01096 | 0.01225 | 0.01389 | 0.01593 |
| 0.4 | 0.00789 | 0.00803 | 0.00829 | 0.00868 | 0.00921 | 0.00993 | 0.01087 | 0.01207 | 0.01359 | 0.01549 |
| 0.5 | 0.00799 | 0.00812 | 0.00835 | 0.00870 | 0.00918 | 0.00983 | 0.01068 | 0.01176 | 0.01313 | 0.01483 |
| 0.6 | 0.00807 | 0.00819 | 0.00839 | 0.00869 | 0.00911 | 0.00967 | 0.01039 | 0.01132 | 0.01250 | 0.01396 |
| 0.7 | 0.00815 | 0.00824 | 0.00840 | 0.00865 | 0.00898 | 0.00943 | 0.01001 | 0.01076 | 0.01170 | 0.01287 |
| 0.8 | 0.00821 | 0.00828 | 0.00840 | 0.00857 | 0.00881 | 0.00913 | 0.00954 | 0.01007 | 0.01074 | 0.01157 |
| 0.9 | 0.00828 | 0.00831 | 0.00837 | 0.00847 | 0.00859 | 0.00876 | 0.00898 | 0.00926 | 0.00962 | 0.01005 |

$D_1$=(v=6,$b_1$=6,$r_1$=5,$k_1$=5,$\lambda_1$=4);$D_2$=(v=6,$b_2$=15,$r_2$=5,$k_2$=2,$\lambda_2$=1),N=312

| **$\rho$** | **d=0.1** | **d=0.2** | **d=0.3** | **d=0.4** | **d=0.5** | **d=0.6** | **d=0.7** | **d=0.8** | **d=0.9** | **d=1** |
|---|---|---|---|---|---|---|---|---|---|---|
| -0.9 | 0.00364 | 0.00362 | 0.00357 | 0.00352 | 0.00347 | 0.00342 | 0.00341 | 0.00343 | 0.00351 | 0.00368 |
| -0.8 | 0.00472 | 0.00471 | 0.00470 | 0.00470 | 0.00472 | 0.00479 | 0.00493 | 0.00515 | 0.00550 | 0.00601 |
| -0.7 | 0.00524 | 0.00525 | 0.00529 | 0.00536 | 0.00548 | 0.00567 | 0.00598 | 0.00642 | 0.00705 | 0.00791 |
| -0.6 | 0.00554 | 0.00559 | 0.00567 | 0.00580 | 0.00602 | 0.00634 | 0.00680 | 0.00745 | 0.00834 | 0.00951 |
| -0.5 | 0.00575 | 0.00581 | 0.00594 | 0.00613 | 0.00643 | 0.00687 | 0.00748 | 0.00831 | 0.00941 | 0.01087 |
| -0.4 | 0.00589 | 0.00598 | 0.00614 | 0.00639 | 0.00677 | 0.00730 | 0.00802 | 0.00901 | 0.01031 | 0.01200 |
| -0.3 | 0.00600 | 0.00611 | 0.00630 | 0.00660 | 0.00703 | 0.00764 | 0.00848 | 0.00958 | 0.01104 | 0.01292 |
| -0.2 | 0.00608 | 0.00621 | 0.00642 | 0.00676 | 0.00724 | 0.00791 | 0.00883 | 0.01004 | 0.01161 | 0.01363 |
| -0.1 | 0.00615 | 0.00628 | 0.00652 | 0.00688 | 0.00740 | 0.00812 | 0.00909 | 0.01037 | 0.01202 | 0.01413 |
| 0 | 0.00621 | 0.00635 | 0.00659 | 0.00697 | 0.00752 | 0.00826 | 0.00926 | 0.01058 | 0.01228 | 0.01444 |
| 0.1 | 0.00625 | 0.00639 | 0.00665 | 0.00703 | 0.00758 | 0.00834 | 0.00935 | 0.01067 | 0.01237 | 0.01454 |
| 0.2 | 0.00629 | 0.00643 | 0.00668 | 0.00707 | 0.00761 | 0.00836 | 0.00935 | 0.01065 | 0.01232 | 0.01443 |
| 0.3 | 0.00632 | 0.00645 | 0.00670 | 0.00705 | 0.00760 | 0.00832 | 0.00927 | 0.01051 | 0.01211 | 0.01413 |
| 0.4 | 0.00634 | 0.00647 | 0.00670 | 0.00705 | 0.00754 | 0.00822 | 0.00911 | 0.01022 | 0.01175 | 0.01362 |
| 0.5 | 0.00636 | 0.00648 | 0.00669 | 0.00700 | 0.00745 | 0.00805 | 0.00886 | 0.00999 | 0.01124 | 0.01292 |
| 0.6 | 0.00638 | 0.00648 | 0.00666 | 0.00693 | 0.00732 | 0.00784 | 0.00853 | 0.00943 | 0.01057 | 0.01202 |
| 0.7 | 0.00639 | 0.00647 | 0.00662 | 0.00684 | 0.00715 | 0.00757 | 0.00812 | 0.00884 | 0.00976 | 0.01991 |
| 0.8 | 0.00640 | 0.00645 | 0.00656 | 0.00671 | 0.00693 | 0.00723 | 0.00762 | 0.00814 | 0.00878 | 0.00960 |
| 0.9 | 0.00640 | 0.00643 | 0.00649 | 0.00657 | 0.00668 | 0.00685 | 0.00705 | 0.00732 | 0.00767 | 0.00811 |

$D_1=(v=7,b_1=7,r_1=4,k_1=4,\lambda_1=2);D_2=(v=7,b_2=21,r_2=6,k_2=2,\lambda_2=1),N=392$

| ρ | d=0.1 | d=0.2 | d=0.3 | d=0.4 | d=0.5 | d=0.6 | d=0.7 | d=0.8 | d=0.9 | d=1 |
|---|---|---|---|---|---|---|---|---|---|---|
| -0.9 | 0.00428 | 0.00421 | 0.00410 | 0.00398 | 0.00386 | 0.00379 | 0.00380 | 0.00394 | 0.00421 | 0.00485 |
| -0.8 | 0.00468 | 0.00466 | 0.00464 | 0.00465 | 0.00473 | 0.00494 | 0.00534 | 0.00600 | 0.00702 | 0.00849 |
| -0.7 | 0.00484 | 0.00486 | 0.00493 | 0.00508 | 0.00536 | 0.00584 | 0.00661 | 0.00776 | 0.00940 | 0.01168 |
| -0.6 | 0.00493 | 0.00500 | 0.00515 | 0.00542 | 0.00588 | 0.00660 | 0.00769 | 0.00927 | 0.01148 | 0.01446 |
| -0.5 | 0.00499 | 0.00510 | 0.00532 | 0.00570 | 0.00631 | 0.00725 | 0.00863 | 0.01058 | 0.01326 | 0.01684 |
| -0.4 | 0.00503 | 0.00518 | 0.00546 | 0.00593 | 0.00667 | 0.00779 | 0.00940 | 0.01166 | 0.01474 | 0.01883 |
| -0.3 | 0.00506 | 0.00524 | 0.00557 | 0.00612 | 0.00697 | 0.00823 | 0.01003 | 0.01254 | 0.01593 | 0.02041 |
| -0.2 | 0.00509 | 0.00528 | 0.00565 | 0.00626 | 0.00720 | 0.00857 | 0.01052 | 0.01321 | 0.01683 | 0.02160 |
| -0.1 | 0.00510 | 0.00532 | 0.00572 | 0.00637 | 0.00736 | 0.00881 | 0.01085 | 0.01367 | 0.01744 | 0.02240 |
| 0 | 0.00512 | 0.00534 | 0.00576 | 0.00644 | 0.00746 | 0.00895 | 0.01105 | 0.01392 | 0.01776 | 0.02280 |
| 0.1 | 0.00513 | 0.00536 | 0.00578 | 0.00647 | 0.00750 | 0.00900 | 0.01109 | 0.01396 | 0.01779 | 0.02281 |
| 0.2 | 0.00514 | 0.00536 | 0.00579 | 0.00646 | 0.00748 | 0.00895 | 0.01101 | 0.01381 | 0.01755 | 0.02242 |
| 0.3 | 0.00514 | 0.00536 | 0.00577 | 0.00642 | 0.00740 | 0.00880 | 0.01077 | 0.01344 | 0.01699 | 0.02163 |
| 0.4 | 0.00514 | 0.00535 | 0.00573 | 0.00634 | 0.00726 | 0.00858 | 0.01039 | 0.01287 | 0.01617 | 0.02046 |
| 0.5 | 0.00514 | 0.00533 | 0.00568 | 0.00623 | 0.00704 | 0.00823 | 0.00987 | 0.01209 | 0.01504 | 0.01889 |
| 0.6 | 0.00514 | 0.00530 | 0.00560 | 0.00607 | 0.00678 | 0.00779 | 0.00920 | 0.01110 | 0.01363 | 0.01692 |
| 0.7 | 0.00513 | 0.00526 | 0.00550 | 0.00588 | 0.00645 | 0.00726 | 0.00839 | 0.00991 | 0.01193 | 0.01485 |
| 0.8 | 0.00512 | 0.00521 | 0.00538 | 0.00566 | 0.00606 | 0.00663 | 0.00743 | 0.00851 | 0.00994 | 0.01180 |
| 0.9 | 0.00511 | 0.00516 | 0.00525 | 0.00539 | 0.00561 | 0.00591 | 0.00634 | 0.00691 | 0.00766 | 0.00864 |

$D_1=(v=8,b_1=14,r_1=7,k_1=4,\lambda_1=3);D_2=(v=8,b_2=28,r_2=7,k_2=2,\lambda_2=1),N=672$

| ρ | d=0.1 | d=0.2 | d=0.3 | d=0.4 | d=0.5 | d=0.6 | d=0.7 | d=0.8 | d=0.9 | d=1 |
|---|---|---|---|---|---|---|---|---|---|---|
| -0.9 | 0.00206 | 0.00204 | 0.00201 | 0.00198 | 0.00198 | 0.00202 | 0.00214 | 0.00236 | 0.00272 | 0.00326 |
| -0.8 | 0.00246 | 0.00247 | 0.00249 | 0.00255 | 0.00267 | 0.00290 | 0.00328 | 0.00387 | 0.00472 | 0.00591 |
| -0.7 | 0.00264 | 0.00267 | 0.00275 | 0.00290 | 0.00315 | 0.00357 | 0.00420 | 0.00513 | 0.00644 | 0.00821 |
| -0.6 | 0.00274 | 0.00280 | 0.00293 | 0.00316 | 0.00354 | 0.00412 | 0.00499 | 0.00623 | 0.00794 | 0.01023 |
| -0.5 | 0.00281 | 0.00290 | 0.00307 | 0.00338 | 0.00386 | 0.00459 | 0.00565 | 0.00715 | 0.00921 | 0.01195 |
| -0.4 | 0.00285 | 0.00297 | 0.00318 | 0.00355 | 0.00412 | 0.00497 | 0.00620 | 0.00792 | 0.01027 | 0.01337 |
| -0.3 | 0.00289 | 0.00302 | 0.00327 | 0.00368 | 0.00433 | 0.00528 | 0.00665 | 0.00854 | 0.01111 | 0.01451 |
| -0.2 | 0.00292 | 0.00306 | 0.00334 | 0.00379 | 0.00449 | 0.00552 | 0.00698 | 0.00901 | 0.01175 | 0.01535 |
| -0.1 | 0.00294 | 0.00310 | 0.00339 | 0.00387 | 0.00460 | 0.00568 | 0.00722 | 0.00933 | 0.01217 | 0.01591 |
| 0 | 0.00296 | 0.00312 | 0.00343 | 0.00392 | 0.00467 | 0.00578 | 0.00734 | 0.00950 | 0.01238 | 0.01617 |
| 0.1 | 0.00297 | 0.00313 | 0.00344 | 0.00394 | 0.00469 | 0.00581 | 0.00737 | 0.00951 | 0.01238 | 0.01615 |
| 0.2 | 0.00298 | 0.00314 | 0.00344 | 0.00394 | 0.00468 | 0.00577 | 0.00729 | 0.00938 | 0.01218 | 0.01584 |
| 0.3 | 0.00298 | 0.00314 | 0.00343 | 0.00391 | 0.00462 | 0.00565 | 0.00711 | 0.00910 | 0.01176 | 0.01524 |
| 0.4 | 0.00299 | 0.00314 | 0.00341 | 0.00385 | 0.00451 | 0.00548 | 0.00683 | 0.00867 | 0.01113 | 0.01435 |
| 0.5 | 0.00299 | 0.00312 | 0.00337 | 0.00377 | 0.00436 | 0.00523 | 0.00644 | 0.00809 | 0.01030 | 0.01318 |
| 0.6 | 0.00331 | 0.00311 | 0.00332 | 0.00366 | 0.00417 | 0.00491 | 0.00595 | 0.00737 | 0.00925 | 0.01171 |
| 0.7 | 0.00299 | 0.00308 | 0.00325 | 0.00353 | 0.00394 | 0.00453 | 0.00536 | 0.00649 | 0.00799 | 0.00996 |
| 0.8 | 0.00298 | 0.00305 | 0.00317 | 0.00337 | 0.00366 | 0.00408 | 0.00466 | 0.00547 | 0.00653 | 0.00792 |
| 0.9 | 0.00298 | 0.00301 | 0.00308 | 0.00318 | 0.00334 | 0.00356 | 0.00387 | 0.00429 | 0.00485 | 0.00559 |

$D_1=(v=10,b_1=18,r_1=9,k_1=45,\lambda_1=4);D_2=(v=10,b_2=45,r_2=9,k_2=2,\lambda_2=1),N=936$

| ρ | d=0.1 | d=0.2 | d=0.3 | d=0.4 | d=0.5 | d=0.6 | d=0.7 | d=0.8 | d=0.9 | d=1 |
|---|---|---|---|---|---|---|---|---|---|---|
| -0.9 | 0.00099 | 0.00100 | 0.00101 | 0.00104 | 0.00111 | 0.00121 | 0.00138 | 0.00164 | 0.00200 | 0.00250 |
| -0.8 | 0.00139 | 0.00141 | 0.00146 | 0.00156 | 0.00172 | 0.00196 | 0.00234 | 0.00287 | 0.00362 | 0.00462 |
| -0.7 | 0.00160 | 0.00165 | 0.00174 | 0.00190 | 0.00215 | 0.00254 | 0.00310 | 0.00390 | 0.00499 | 0.00646 |
| -0.6 | 0.00174 | 0.00180 | 0.00193 | 0.00215 | 0.00249 | 0.00301 | 0.00375 | 0.00478 | 0.00619 | 0.00805 |
| -0.5 | 0.00183 | 0.00191 | 0.00208 | 0.00235 | 0.00277 | 0.00339 | 0.00428 | 0.00552 | 0.00720 | 0.00941 |
| -0.4 | 0.00190 | 0.00200 | 0.00219 | 0.00251 | 0.00299 | 0.00371 | 0.00473 | 0.00613 | 0.00803 | 0.01053 |
| -0.3 | 0.00195 | 0.00207 | 0.00228 | 0.00263 | 0.00317 | 0.00396 | 0.00508 | 0.00662 | 0.00869 | 0.01142 |
| -0.2 | 0.00199 | 0.00212 | 0.00235 | 0.00273 | 0.00331 | 0.00416 | 0.00535 | 0.00699 | 0.00919 | 0.01207 |
| -0.1 | 0.00203 | 0.00216 | 0.00240 | 0.00280 | 0.00341 | 0.00429 | 0.00553 | 0.00723 | 0.00951 | 0.01249 |
| 0 | 0.00205 | 0.00219 | 0.00244 | 0.00285 | 0.00309 | 0.00437 | 0.00563 | 0.00736 | 0.00967 | 0.01269 |
| 0.1 | 0.00207 | 0.00221 | 0.00246 | 0.00287 | 0.00349 | 0.00439 | 0.00564 | 0.00736 | 0.00966 | 0.01266 |
| 0.2 | 0.00209 | 0.00222 | 0.00247 | 0.00287 | 0.00348 | 0.00435 | 0.00558 | 0.00725 | 0.00948 | 0.01240 |
| 0.3 | 0.00210 | 0.00223 | 0.00247 | 0.00285 | 0.00343 | 0.00426 | 0.00543 | 0.00702 | 0.00914 | 0.01191 |
| 0.4 | 0.00211 | 0.00223 | 0.00246 | 0.00281 | 0.00335 | 0.00412 | 0.00520 | 0.00667 | 0.00864 | 0.01120 |
| 0.5 | 0.00212 | 0.00223 | 0.00243 | 0.00275 | 0.00305 | 0.00392 | 0.00489 | 0.00621 | 0.00797 | 0.01026 |
| 0.6 | 0.00213 | 0.00222 | 0.00239 | 0.00267 | 0.00308 | 0.00367 | 0.00450 | 0.00563 | 0.00713 | 0.00909 |
| 0.7 | 0.00213 | 0.00221 | 0.00234 | 0.00256 | 0.00289 | 0.00337 | 0.00403 | 0.00493 | 0.00612 | 0.00769 |
| 0.8 | 0.00213 | 0.00219 | 0.00228 | 0.00244 | 0.00267 | 0.00301 | 0.00347 | 0.00411 | 0.00495 | 0.00606 |
| 0.9 | 0.00213 | 0.00216 | 0.00221 | 0.00229 | 0.00242 | 0.00259 | 0.00284 | 0.00318 | 0.00362 | 0.00420 |

$D_1=(v=7,b_1=7,r_1=4,k_1=4,\lambda_1=2);D_2=(v=7,b_2=21,r_2=6,k_2=2,\lambda_2=1),N=392$

| ρ | d=0.1 | d=0.2 | d=0.3 | d=0.4 | d=0.5 | d=0.6 | d=0.7 | d=0.8 | d=0.9 | d=1 |
|---|---|---|---|---|---|---|---|---|---|---|
| -0.9 | 0.00089 | 0.00089 | 0.00088 | 0.00087 | 0.00087 | 0.00088 | 0.00091 | 0.00097 | 0.00106 | 0.00119 |
| -0.8 | 0.00092 | 0.00092 | 0.00094 | 0.00096 | 0.00101 | 0.00108 | 0.00119 | 0.00136 | 0.00160 | 0.00193 |
| -0.7 | 0.00093 | 0.00095 | 0.00098 | 0.00103 | 0.00111 | 0.00124 | 0.00144 | 0.00171 | 0.00208 | 0.00258 |
| -0.6 | 0.00094 | 0.00096 | 0.00101 | 0.00109 | 0.00121 | 0.00139 | 0.00165 | 0.00201 | 0.00249 | 0.00314 |
| -0.5 | 0.00094 | 0.00098 | 0.00104 | 0.00113 | 0.00129 | 0.00151 | 0.00183 | 0.00227 | 0.00285 | 0.00363 |
| -0.4 | 0.00095 | 0.00099 | 0.00106 | 0.00117 | 0.00135 | 0.00161 | 0.00197 | 0.00247 | 0.00314 | 0.00400 |
| -0.3 | 0.00095 | 0.00099 | 0.00108 | 0.00121 | 0.00140 | 0.00169 | 0.00209 | 0.00264 | 0.00338 | 0.00434 |
| -0.2 | 0.00095 | 0.00100 | 0.00109 | 0.00123 | 0.00144 | 0.00175 | 0.00218 | 0.00277 | 0.00355 | 0.00458 |
| -0.1 | 0.00095 | 0.00100 | 0.00110 | 0.00125 | 0.00147 | 0.00179 | 0.00224 | 0.00285 | 0.00366 | 0.00473 |
| 0 | 0.00096 | 0.00101 | 0.00110 | 0.00126 | 0.00148 | 0.00181 | 0.00227 | 0.00289 | 0.00372 | 0.00479 |
| 0.1 | 0.00096 | 0.00101 | 0.00111 | 0.00126 | 0.00149 | 0.00182 | 0.00227 | 0.00289 | 0.00371 | 0.00478 |
| 0.2 | 0.00096 | 0.00101 | 0.00110 | 0.00125 | 0.00148 | 0.00180 | 0.00224 | 0.00284 | 0.00364 | 0.00468 |
| 0.3 | 0.00096 | 0.00101 | 0.00110 | 0.00124 | 0.00145 | 0.00176 | 0.00218 | 0.00276 | 0.00351 | 0.00450 |
| 0.4 | 0.00095 | 0.00100 | 0.00108 | 0.00122 | 0.00142 | 0.00170 | 0.00209 | 0.00263 | 0.00333 | 0.00424 |
| 0.5 | 0.00095 | 0.00100 | 0.00107 | 0.00119 | 0.00137 | 0.00162 | 0.00198 | 0.00245 | 0.00308 | 0.00390 |
| 0.6 | 0.00095 | 0.00099 | 0.00105 | 0.00116 | 0.00131 | 0.00153 | 0.00182 | 0.00223 | 0.00277 | 0.00347 |
| 0.7 | 0.00095 | 0.00098 | 0.00103 | 0.00111 | 0.00124 | 0.00141 | 0.00165 | 0.00198 | 0.00241 | 0.00297 |
| 0.8 | 0.00095 | 0.00097 | 0.00109 | 0.00106 | 0.00115 | 0.00127 | 0.00145 | 0.00168 | 0.00198 | 0.00237 |
| 0.9 | 0.00094 | 0.00096 | 0.00097 | 0.00101 | 0.00105 | 0.00112 | 0.00121 | 0.00133 | 0.00149 | 0.00170 |

$D_1=(v=13,b_1=26,r_1=12,k_1=6,\lambda_1=5);D_2=(v=13,b_2=13,r_2=4,k_2=4,\lambda_2=1),N=2080$

| ρ | d=0.1 | d=0.2 | d=0.3 | d=0.4 | d=0.5 | d=0.6 | d=0.7 | d=0.8 | d=0.9 | d=1 |
|---|---|---|---|---|---|---|---|---|---|---|
| -0.9 | 0.00050 | 0.00050 | 0.00051 | 0.00052 | 0.00054 | 0.00058 | 0.00063 | 0.00071 | 0.00081 | 0.00096 |
| -0.8 | 0.00067 | 0.00068 | 0.00070 | 0.00073 | 0.00079 | 0.00087 | 0.00099 | 0.00115 | 0.00137 | 0.00168 |
| -0.7 | 0.00076 | 0.00077 | 0.00081 | 0.00087 | 0.00096 | 0.00108 | 0.00127 | 0.00152 | 0.00186 | 0.00230 |
| -0.6 | 0.00081 | 0.00084 | 0.00088 | 0.00096 | 0.00108 | 0.00125 | 0.00149 | 0.00182 | 0.00225 | 0.00282 |
| -0.5 | 0.00085 | 0.00088 | 0.00094 | 0.00103 | 0.00118 | 0.00139 | 0.00168 | 0.00207 | 0.00259 | 0.00327 |
| -0.4 | 0.00087 | 0.00091 | 0.00098 | 0.00109 | 0.00126 | 0.00150 | 0.00183 | 0.00227 | 0.00287 | 0.00364 |
| -0.3 | 0.00089 | 0.00093 | 0.00101 | 0.00114 | 0.00132 | 0.00158 | 0.00194 | 0.00243 | 0.00307 | 0.00391 |
| -0.2 | 0.00091 | 0.00095 | 0.00104 | 0.00117 | 0.00137 | 0.00165 | 0.00204 | 0.00256 | 0.00325 | 0.00414 |
| -0.1 | 0.00092 | 0.00097 | 0.00106 | 0.00120 | 0.00140 | 0.00170 | 0.00210 | 0.00264 | 0.00336 | 0.00428 |
| 0 | 0.00093 | 0.00098 | 0.00107 | 0.00121 | 0.00142 | 0.00172 | 0.00213 | 0.00268 | 0.00341 | 0.00434 |
| 0.1 | 0.00094 | 0.00099 | 0.00108 | 0.00122 | 0.00143 | 0.00173 | 0.00213 | 0.00268 | 0.00339 | 0.00432 |
| 0.2 | 0.00094 | 0.00099 | 0.00108 | 0.00122 | 0.00143 | 0.00172 | 0.00211 | 0.00265 | 0.00334 | 0.00425 |
| 0.3 | 0.00095 | 0.00099 | 0.00108 | 0.00121 | 0.00141 | 0.00168 | 0.00206 | 0.00257 | 0.00323 | 0.00408 |
| 0.4 | 0.00095 | 0.00100 | 0.00107 | 0.00120 | 0.00138 | 0.00164 | 0.00199 | 0.00245 | 0.00307 | 0.00386 |
| 0.5 | 0.00095 | 0.00099 | 0.00106 | 0.00118 | 0.00134 | 0.00157 | 0.00188 | 0.00230 | 0.00285 | 0.00357 |
| 0.6 | 0.00096 | 0.00099 | 0.00105 | 0.00115 | 0.00129 | 0.00148 | 0.00175 | 0.00211 | 0.00258 | 0.00319 |
| 0.7 | 0.00096 | 0.00098 | 0.00103 | 0.00111 | 0.00122 | 0.00138 | 0.00159 | 0.00188 | 0.00225 | 0.00274 |
| 0.8 | 0.00096 | 0.00098 | 0.00101 | 0.00107 | 0.00114 | 0.00125 | 0.00141 | 0.00161 | 0.00187 | 0.00221 |
| 0.9 | 0.00096 | 0.00097 | 0.00099 | 0.00101 | 0.00106 | 0.00111 | 0.00119 | 0.00130 | 0.00144 | 0.00161 |

$D_1=(v=15,b_1=15,r_1=7,k_1=7,\lambda_1=3);D_2=(v=15,b_2=35,r_2=7,k_2=3,\lambda_2=1),N=2480$

| ρ | d=0.1 | d=0.2 | d=0.3 | d=0.4 | d=0.5 | d=0.6 | d=0.7 | d=0.8 | d=0.9 | d=1 |
|---|---|---|---|---|---|---|---|---|---|---|
| -0.9 | 0.00075 | 0.00074 | 0.00074 | 0.00075 | 0.00076 | 0.00078 | 0.00082 | 0.00088 | 0.00098 | 0.00112 |
| -0.8 | 0.00078 | 0.00078 | 0.00080 | 0.00083 | 0.00088 | 0.00096 | 0.00108 | 0.00125 | 0.00149 | 0.00181 |
| -0.7 | 0.00079 | 0.00079 | 0.00084 | 0.00089 | 0.00098 | 0.00111 | 0.00130 | 0.00157 | 0.00193 | 0.00241 |
| -0.6 | 0.00080 | 0.00081 | 0.00087 | 0.00095 | 0.00107 | 0.00124 | 0.00149 | 0.00184 | 0.00231 | 0.00293 |
| -0.5 | 0.00080 | 0.00082 | 0.00089 | 0.00099 | 0.00114 | 0.00136 | 0.00166 | 0.00208 | 0.00265 | 0.00339 |
| -0.4 | 0.00080 | 0.00083 | 0.00100 | 0.00111 | 0.00133 | 0.00165 | 0.00198 | 0.00240 | 0.00310 | 0.00400 |
| -0.3 | 0.00080 | 0.00083 | 0.00101 | 0.00122 | 0.00141 | 0.00166 | 0.00200 | 0.00260 | 0.00330 | 0.00431 |
| -0.2 | 0.00081 | 0.00083 | 0.00102 | 0.00122 | 0.00140 | 0.00170 | 0.00211 | 0.00274 | 0.00355 | 0.00453 |
| -0.1 | 0.00082 | 0.00084 | 0.00115 | 0.00122 | 0.00140 | 0.00175 | 0.00222 | 0.00287 | 0.00360 | 0.00472 |
| 0 | 0.00082 | 0.00085 | 0.00115 | 0.00120 | 0.00140 | 0.00180 | 0.00220 | 0.00289 | 0.00370 | 0.00470 |
| 0.1 | 0.00090 | 0.00099 | 0.00116 | 0.00120 | 0.00140 | 0.00181 | 0.00220 | 0.00280 | 0.00370 | 0.00470 |
| 0.2 | 0.00090 | 0.00091 | 0.00119 | 0.00120 | 0.00140 | 0.00182 | 0.00220 | 0.00280 | 0.00360 | 0.00460 |
| 0.3 | 0.00093 | 0.00092 | 0.00111 | 0.00121 | 0.00141 | 0.00177 | 0.00210 | 0.00270 | 0.00351 | 0.00455 |
| 0.4 | 0.00093 | 0.00092 | 0.00100 | 0.00122 | 0.00142 | 0.00170 | 0.00200 | 0.00260 | 0.00330 | 0.00428 |
| 0.5 | 0.00099 | 0.00095 | 0.00100 | 0.00111 | 0.00133 | 0.00169 | 0.00190 | 0.00240 | 0.00303 | 0.00391 |
| 0.6 | 0.00099 | 0.00105 | 0.00101 | 0.00110 | 0.00132 | 0.00155 | 0.00180 | 0.00220 | 0.00270 | 0.00344 |
| 0.7 | 0.00099 | 0.00100 | 0.00100 | 0.00110 | 0.00120 | 0.00141 | 0.00160 | 0.00190 | 0.00240 | 0.00290 |
| 0.8 | 0.00100 | 0.00100 | 0.00101 | 0.00100 | 0.00111 | 0.00120 | 0.00140 | 0.00160 | 0.00190 | 0.00233 |
| 0.9 | 0.00101 | 0.00100 | 0.00099 | 0.00100 | 0.00101 | 0.00115 | 0.00120 | 0.00130 | 0.00140 | 0.00170 |

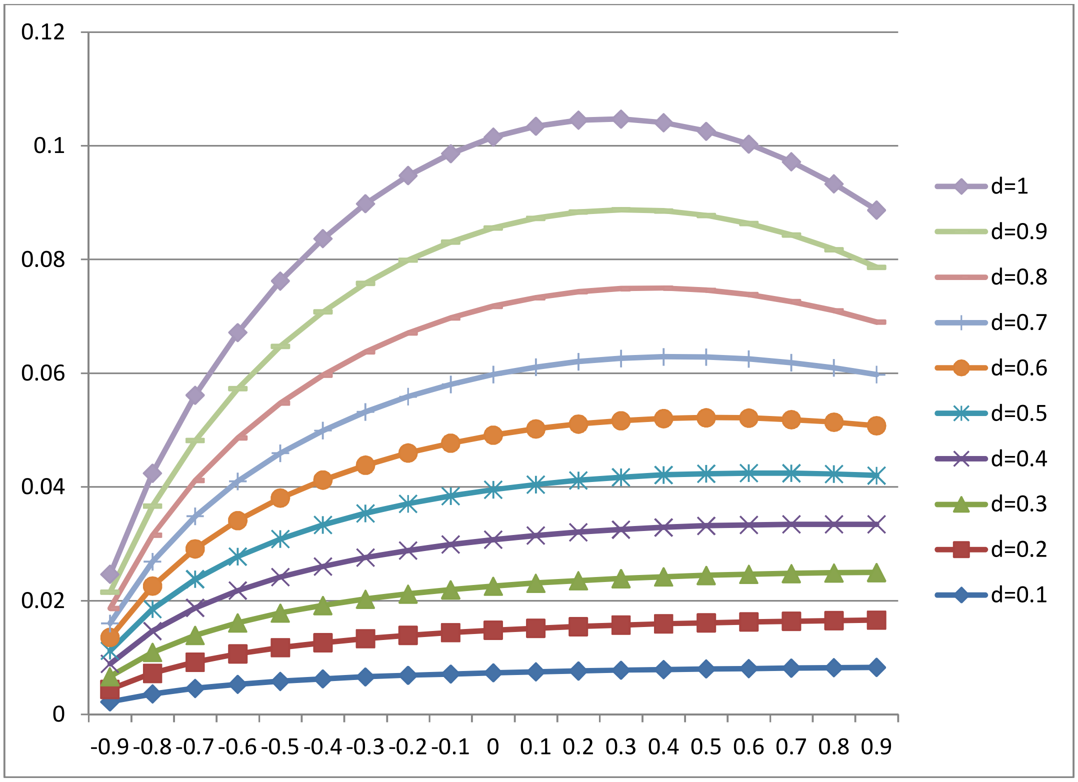


**Fig. 1. Graphical representation for SORD under under tri-diagonal correlated structure of errors using a pair of BIBD with parameters $D_1$=(v=5, $b_1$=5, $r_1$=4, $k_1$=4, $\lambda_1$=3) and $D_2$=(v=5, $b_2$=10, $r_2$=4, $k_2$=2, $\lambda_2$=1) for 5 factors with N=240**

## 6 Conclusions

A new unified method of construction of second order rotatable designs under tri-diagonal correlated structure of errors using two suitably chosen balanced incomplete block designs is suggested.
From Table 1 and Table 2 of Appendix, we observed that

1. $V(\hat{Y}_u)$ is slightly increasing for different values of $\rho$ is increasing.
2. $V(\hat{Y}_u)$ is slightly increasing for taking $d=0.1(0.1)1.0$, for given $v$, $\rho$ .

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## Competing Interests

Authors have declared that no competing interests exist.

## References

[1] Box GEP, Hunter JS. Multifactor experimental designs for exploring response surfaces, Annals of Mathematical Statistics. 1957;28:195-241.

[2] Das MN, Narasimham VL. Construction of rotatable designs through balanced incomplete block designs, Annals of Mathematical Statistics. 1962;33:1421-1439.

[3] Narasimham VL, Ramachandrarao P, Rao KN. Construction of second order rotatable designs through a pair of balanced incomplete block designs, Journal of the Indian Society Agricultural Statistics. 1983;35:36-40.

[4] Panda RN, Das RN. First order rotatable designs with correlated errors. Calcutta Statistical Association Bulletin. 1994;44:83-101.

[5] Das RN. Robust second order rotatable designs: Part-I, (RSORD) Calcutta Statistical Association Bulletin. 1997;47:199-214.

[6] Das RN. Robust second order rotatable designs: Part-II, Calcutta Statistical Association Bulletin. 1999;49:65-78.

[7] Das RN. Robust second order rotatable designs: Part-III, Calcutta Statistical Association Bulletin. 2003;56:117-130.

[8] Rajyalakshmi K. Some contributions to second order rotatable and slope rotatable designs under different correlated error structures, unpublished Ph.D thesis, Acharya Nagarjuna University, Guntur-522510, India; 2014.

[9] Rajyalakshmi K. Victorbabu B. Re. Construction of second order rotatable designs under tri-diagonal correlated structure of errors using central composite designs, Journal of Statistics Advances in Theory and Applications. 2014;11:71-90.

[10] Rajyalakshmi K, Victorbabu B. Re. An empirical study of second order rotatable designs under tri-diagonal correlated structure of errors using incomplete block designs, Sri Lankan Journal of Applied Statistics. 2016;17:1-17.

[11] Rajyalakshmi K, Victorbabu B. Re. A note on second order rotatable designs under tri-diagonal correlated structure of errors using balanced incomplete block designs, International Journal of Agricultural and Statistical Sciences. 2018;14:1-4.

[12] Rajyalakshmi K, Prasanthi V, Victorbabu B. Re. An empirical study of Second order rotatable designs under tri-diagonal correlated structure of errors using partially balanced incomplete block type designs. Thailand Statistician. 2020;18(2):122-134.

[13] Sulochana B, Victorbabu B. Re. Second order slope rotatable designs under tri-diagonal correlated structure of errors using a pair of incomplete block designs. Asian Journal of Probability and Statistics. 2020a; 6:1-12.

[14] Ravindrababu M, Victorbabu B. Re. An empirical study on second order rotatable designs under intra class correlated structure of errors using a pair of balanced incomplete block designs. International Journal of Agricultural and Statistical Sciences. 2018;14:775-779.

[15] Ravindrababu M, Victorbabu B. Re. An empirical study on second order rotatable designs under intra class correlated structure of errors using a pair symmetrical unequal block arrangements with two unequal block sizes, International Journal of Agricultural and Statistical Sciences. 2019;15.

[16] Rajyalakshmi K, Victorbabu B. Re. Second order slope rotatable designs under intra-class correlated error structure using balanced incomplete block designs. Thailand Statistician. 2015;13(2):169-183.

[17] Sulochana B, Victorbabu B. Re. A study of second order slope rotatable designs under intra-class correlated structure of errors using a pair of balanced incomplete block designs. Andhra Agricultural Journal. 2019;66:12-20.

[18] Sulochana B, Victorbabu B. Re. A study of second order slope rotatable designs under intra- class correlated structure of errors using partially balanced incomplete block type designs. Asian Journal of Probability and Statistics. 2020;7(4):15-28.

[19] Das RN. Robust response surfaces, Regression, and Positive data analysis, CRC Press, Taylor and Francis Group, New York; 2014.



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