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Experimental Comparisons of Metaheuristic Algorithms in Solving Combined Economic Emission Dispatch Problem Using Parametric and Non-Parametric Tests

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ABSTRACT

In this paper, the parametric and non-parametric statistical tests are applied for comparisons of metaheuristic algorithms' (MAs) behavior in solving Combined Economic Emission Dispatch (CEED) problem. In the last years, in many published papers, a large number of MAs have been proposed to solve CEED problem of different dimensions consisting of different objective functions. In this paper, the statistical tests are applied over samples of results obtained for eight objective functions of CEED problem using the four MAs: Firefly Algorithm, Moth Swarm Algorithm, Adaptive Wind Driven Optimization and Particle Swarm Optimization-Gravitational Search Algorithm. The standard IEEE 30-bus six-generator test system is used. The statistical tests are applied over results obtained for each function and over results obtained for all eight functions simultaneously. The analysis of the results of statistical tests over a single function shows that one MA statistically behaves differently for different functions and one MA is not the best for each function of CEED problem. Therefore, more MAs are more acceptable than one MA for solving a specified CEED problem. However, the analysis of the results of statistical tests over all functions simultaneously shows that all four MAs statistically behave in the same way.

Introduction

In the last years, a number of optimization metaheuristic algorithms (MAs) have been proposed in literature for solving problems in the science, technology, economics, industry, operational research and other fields. For optimization of non-smooth and non-convex functions, that often describe these problems, it is difficult or impossible to use the exact gradient methods and in these cases stochastic MAs are successfully applied. Optimization objectives are different: minimizing losses and energy costs, maximizing efficiency, profit, performance, and outputs. In most cases, the ultimate goal is to obtain

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maximal production or profit based on limited resources, the amount of money and time (Kaveh 2014). The Combined Economic Emission Dispatch (CEED) problem is one of the problems for whose solution in recent times has been proposed a number of MAs. The CEED problem is a key problem in the planning and operation of an electric power system, in which the fuel cost or simultaneously fuel cost and emission of pollutants (SO_2, CO_2 and NO_x) in thermal power plants are minimized. The minimization is done by adjusting the output powers of the generators, so that all the demands and constraints in the system are satisfied. The objective functions are complex and consist of sums of quadratic, sinusoidal and exponential functions. The authors of the paper (Jevtic et al. 2017) provide an overview of over 30 MAs proposed in various published papers for solving the CEED problem. Some of these MAs are: genetic algorithm, artificial bee colony algorithm, particle swarm optimization, differential evolution, gravitational search algorithm, krill herd algorithm, Firefly Algorithm (FA), grey wolf optimizer, flower pollination algorithm, spiral optimization algorithm, nondominated sorting genetic algorithm and nondominated sorting genetic algorithm – ii, niched pareto genetic algorithm, multiobjective particle swarm optimization, modified bacterial foraging algorithm, hybrid multi-objective optimization algorithm based on particle swarm optimization and differential evolution, hybrid differential evolution, elitist multiobjective evolutionary algorithm, biogeography-based optimization algorithm, multi-objective θ-particle swarm optimization, opposition-based gravitational search algorithm, parallel particle swarm optimization algorithm, tribe-modified differential evolution algorithm, fuzzified multi-objective interactive honey-bees mating optimization, multi-objective adaptive clonal selection algorithm, enhanced probability-selection, hybrid Particle Swarm Optimization and Gravitational Search Algorithm (PSOGSA), multi-objective bacterial foraging algorithm, galaxy-based search algorithm, artificial bee colony algorithm with dynamic population size, symbiotic organisms search optimization algorithm, quasi-oppositional group search optimization, self-adaptive firefly algorith, multi-objective hybrid evolutionary algorithm, multi-objective differential evolution algorithm with ensemble of selection, Moth Swarm Algorithm (MSA) and Adaptive Wind Driven Optimization (AWDO). The authors of published papers proposing the MAs for solving the CEED problems use various standard power test systems, different variants of the CEED problem mathematical model and different dimensions of the problem (different number of generators in the system). The authors compared the proposed MAs with other published MAs mainly based on best, average and standard deviation values and convergence profiles of a set of runs over a single problem which is solved. The CEED problem is generally solved for various cases: various numbers of generators in the system (small, medium and large number of generators); minimization of fuel cost; minimization of emission of pollutant;

minimization of fuel cost and emission simultaneously; with and withouth losses in the transmission network; with and without the valve point effect in a thermal power plant. For different cases, the objective function has a different form, and different algorithms generally give different solutions. This is in accordance with the "No free lunch" theorem (Sörensen 2015; Wolpert and Macready 1997) from which follows that it is not possible to find one optimization algorithm being better in behavior for any problem, and it is necessary to know the problem that is solved in order to design the algorithm with appropriate characteristics. In the published papers, the parametric and non-parametric statistical tests were not applied in solving the CEED problem. However, in Demšar (2006); Garcia et al. (2010, 2009), the procedures for use of parametric and non-parametric tests are proposed for comparing and analyzing the performance of MAs on a single data set and on a multiple data set in solving problems in the field of computational intelligence. In Garcia et al. (2009), a study on the use of non-parametric tests for analyzing the evolutionary algorithms' behavior has been done. In addition, in Garcia et al. (2009), the procedures for comparison of evolutionary algorithms were applied on standard benchmark functions. Based on these tests, the authors decided whether there are statistical differences between the algorithms and whether the differences are real or random. In this paper, we use the parametric and non-parametric statistical tests for analysis of the results obtained by using the MAs in solving CEED problem. For this analysis, we selected the four MAs that, based on published papers (Jevtić et al. 2017; Radosavljević 2016; Radosavljević et al. 2015), have the best results in solving CEED problem applying different standard test systems. We perform comparisons of statistical behaviors of selected algorithms using two types of analysis:

- analysis of the results obtained for each function of CEED problem independently.
- analysis of the results obtained for all functions simultaneously.

CEED Functions to Be Minimized

Table 1 shows the eight most common functions that describe the CEED problem and that will be minimized. The constraints are:

(1) the power equality constraint in the transmission system:

$$\sum_{g \in G} P_g - P_D - P_{loss} = 0, \tag{1}$$

where P_{loss} is power loss in the system and P_D is total load demand.

Table 1. Description of the CEED functions to be minimized.

Description of functions f1–f8	
f1	$F(P_g) = \sum_{g\in G}\left(a_g + b_g P_g + c_g P_g^2\right), g = 1, 2, \ldots, G$ $F(P_g)$ is the fuel cost function of each generator in thermal power plants (\$/h); P_g is the output power of generator g (MW); G is the total number of generators; a_g, b_g and c_g are the cost coefficients.
f2	$E(P_g) = \sum_{g\in G}\left(\alpha_g + \beta_g P_g + \eta_g P_g^2 + \xi_g \exp(\lambda_g P_g)\right)$ $E(P_g)$ is the emission function of each generator in thermal power plants (t/h); α_g, β_g, η_g, ξ_g and λ_g are the emission coefficients of the generation unit g.
f3	$FE(P_g) = F(P_g) + \gamma E(P_g)$ $FE(P_g)$ is a combined function obtained by the sum of previous functions; γ is the scaling factor.
f4	$F_{loss}(P_g) = F(P_g) + \lambda_p\left(P_G - P_G^{\lim}\right)^2$ $F_{loss}(P_g)$ is a function consisting of $F(P_g)$ and quadratic penalty term, which depends on power loss in the system (Jevtić et al. 2017); λ_p is the penalty factor.
f5	$FE_{loss}(P_g) = F(P_g) + \gamma E(P_g) + \lambda_p\left(P_G - P_G^{\lim}\right)^2$ $FE_{loss}(P_g)$ is a combined function which takes into account the power loss.
f6	$F_{VPE}(P_g) = F(P_g) + \sum_{g\in G}\left\lvert d_g \sin\left(e_g\left(P_g^{\min} - P_g\right)\right)\right\rvert + \lambda_p\left(P_G - P_G^{\lim}\right)^2$ $F_{VPE}(P_g)$ is the fuel cost function which takes into account the power loss and valve point effect (VPE) in the thermal power plant; d_g and e_g are coefficients for VPLE.
f7	$E_{loss}(P_g) = E(P_g) + \lambda_p\left(P_G - P_G^{\lim}\right)^2$ $E_{loss}(P_g)$ is the emission function which takes into account the power loss.
f8	$FE_{VPE}(P_g) = F(P_g) + \sum_{g\in G}\left\lvert d_g \sin\left(e_g\left(P_g^{\min} - P_g\right)\right)\right\rvert + \gamma E(P_g) + \lambda_p\left(P_G - P_G^{\lim}\right)^2$ $FE_{VPE}(P_g)$ is a combined function which takes into account the power loss and VPLE

(2) the generator capacity constraint:

$$P_g^{\min} \le P_g \le P_g^{\max}, \tag{2}$$

where $P_g^{\min}$ and $P_g^{\max}$are minimal and maximal generator power, respectively.

The power loss of the transmission system is expressed using *B-loss* matrices, as follows (Aydin et al. 2014):

$$P_{loss} = \sum\nolimits_{g\in G}\sum\nolimits_{j\in G} P_g B_{gj} P_j + \sum\nolimits_{g\in G} B_{0g} P_g + B_{00}\,, \tag{3}$$

where B_{00}, B_{0g} and B_{gj} are the coefficients of the *B-loss* matrices. The value of $P_g^{\lim}$ is computed during the optimization process in order to satisfy the power equality constraint in the system and the calculation procedure is described in detail in the paper (Jevtić et al. 2017).

Experimentation

In this paper, we carry out a statistical analysis of four MAs that gave the best results, compared to other MAs, in solving the CEED problem for test systems (Jevtić et al. 2017; Radosavljević 2016). These algorithms are: AWDO (Bayraktar and Komurcu 2015), PSOGSA (Mirjalli, Hashim, and Sardroudi 2012), MSA (Mohamed et al. 2017) and FA (Yang 2010). We carry out the minimization of the eight functions listed in Table 1. The power

Table 2. The coefficients of algorithms that apply to the test system.

AWDO			MSA			FA					PSOGSA					
N	T	α, g, RT, c	N	T	N_c	N	T	α	β_{min}	γ	N	T	G_0	α	C_1	C_2
50	200	Optimized	50	200	6	50	200	0.25	0.2	1	50	200	1	10	2	2

system in which the CEED problem is solved is the standard IEEE 30-bus six-generators system with a total load demand of 283.4 MW. The *B-loss* matrices, the emission coefficients and fuel cost coefficients are taken from Aydin et al. (2014). The algorithms are implemented in MATLAB 2011b computational environment on a notebook at 2.2 GHz, 3.0 GB RAM. The coefficients of the update equations for the simulations are fine-tuned and presented in Table 2. The AWDO eliminates the need for tuning the coefficients and optimizes the selection of the coefficients at each iteration. The results of the simulations are obtained after 30 runs. Table 3 shows the obtained minimum and mean values, standard deviations and average error rates for all four algorithms and all eight functions. The error rate is calculated as a percentage value of single result in relation to the best value. The average error rate is calculated as the mean of the error rates for each function. The average error rate is considered as a means for measuring the performance of each algorithm (Garcia et al. 2009). The statistical analysis of the behavior of MAs in this paper is carried out using the following tests:

(1) tests of normality of Kolmogorov-Smirnov, Shapiro-Wilk and D'Agostino-Pearson applied over a results obtained for each function independently.
(2) test of heteroscedasticity of Levene.
(3) paired t-test and Wilcoxon test applied over results obtained for each function.
(4) tests of normality of Kolmogorov-Smirnov, Shapiro-Wilk and D'Agostino-Pearson applied over results obtained for all functions simultaneously.
(5) Wilcoxon's and Friedman's non-parametric tests for multiple-problem analysis.

The statistical analysis in this paper is performed by using the statistical software package SPSS.

Numerical Results

At the beginning, normality and heteroscedasticity tests are carried out to determine whether parametric tests can be applied to the behavioral analysis of algorithms in solving the CEED problem. Three normality tests were

Table 3. Best, mean, SD and average error rate values of the results, obtained by using the AWDO, MSA, PSOGSA and FA for the test system (Case I).

Function	Values	AWDO	MSA	PSOGSA	FA
f1	Best	600.111408	600.111408	600.111408	600.111408
	Mean	600.159978	600.111417	600.111408	600.115307
	Std.dev	0.075212	1.23379E-05	4.58952E-08	0.021351
	Average error rate	8.093470E-03	1.53714E-06	1.73065E-08	0.000650
f2	Best	0.194203	0.194203	0.194203	0.194203
	Mean	0.194221	0.194203	0.194203	0.194203
	Std.dev	2.54834E-05	2.87182E-09	4.54627E-11	5.57591E-09
	Average error rate	0.009547	1.48716E-06	4.61545E-08	5.51485E-07
f3	Best	405.043458	405.043458	405.043458	405.043458
	Mean	405.066663	405.043462	405.043458	405.043458
	Std.dev	0.038901	3.7282E-06	5.60405E-08	9.60373E-08
	Average error rate	0.005729	9.47535E-07	2.33044E-08	1.92368E-08
f4	Best	605.998369	605.998370	605.998369	605.998369
	Mean	606.023600	605.998381	618.405798	605.998401
	Std.dev	0.033900	1.5047E-05	10.413550	0.000171
	Average error rate	0.004163	1.87235E-06	2.047436	5.20071E-06
f5	Best	407.911455	407.911455	407.911455	407.911455
	Mean	407.924636	407.911458	420.768338	407.911455
	Std.dev	0.025912	2.60421E-06	9.153432	9.13352E-08
	Average error rate	0.003231	7.97712E-07	3.151881	2.15069E-08
f6	Best	635.819624	635.875068	635.820110	635.869070
	Mean	638.565176	644.427547	661.440545	640.975784
	Std.dev	5.366131	9.453049	15.576525	7.319519
	Average error rate	0.431736	1.353753	4.029510	0.810870
f7	Best	194.178511	194.178511	194.178511	194.178511
	Mean	194.188534	194.178514	204.603907	194.178511
	Std.dev	0.020154	2.17712E-06	6.413748	6.2201E-08
	Average error rate	0.005162	1.36402E-06	5.368975	3.41853E-08
f8	Best	430.852030	430.875609	430.852090	430.854329
	Mean	430.959821	431.294850	442.638341	431.226613
	Std.dev	0.117999	0.363465	7.845553	0.748728
	Average error rate	0.025004	0.102764	2.735568	0.086926

applied: Kolmogorov-Smirnov test which compares the accumulated distribution of sample results obtained by algorithm, with the Gaussian distribution; Shapiro-Wilk test which computes the level of symmetry and kurtosis of the distribution curve; D'Agostino-Pearson test which computes the skewness and kurtosis in comparison with the expected Gaussian distribution. Table 4 shows the probability values (p-values) of the normality tests over the results obtained by algorithms. In this paper, we consider that level of significance for p-values is α = 0.05. From Table 4, it is obvious that all of the p-values are less than significance level α, except in case of PSOGSA use over the functions f5–f8 where the p-value is larger than 0.05. This means that the sample results of the AWDO, MSA and FA algorithms do not follow a normal distribution, while the results of PSOGSA follow the normal distribution of functions f5–f8. If the sample size would be larger, it might be expected that PSOGSA follows the normal distribution for each function. The histograms and Q–Q plots (graphical presentations of the quartiles from

Table 4. *p*-values for tests of normality of Kolmogorov-Smirnov, Shapiro-Wilk and D'Agostino-Pearson.

	Kolmogorov-Smirnov								Shapiro-Wilk							
Algorithm	f1	f2	f3	f4	f5	f6	f7	f8	f1	f2	f3	f4	f5	f6	f7	f8
PSO-GSA	.200	.076	.200	.012	.200	.200	.200	.185	-	-	-	.000	.216	.112	.252	.059
AWDO	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
MSA	.000	.001	.004	.000	.001	.000	.001	.092	.000	.000	.000	.000	.001	.000	.000	.006
FA	.000	.000	.000	.000	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

	D'Agostino-Pearson							
Algorithm	f1	f2	f3	f4	f5	f6	f7	f8
PSO-GSA	-	.000	-	.343	.636	.506	.504	.708
AWDO	.000	.000	.000	.003	.000	.000	.000	.000
MSA	.000	.010	-	.000	.000	.100	.000	.006
FA	.000	.000	-	.000	.000	.001	.000	.000

data observed and those from the normal distributions) for PSOGSA and AWDO are presented in Figure 1 and confirm the results of normality tests. The results of Levene's test show that the sample variances of algorithms for each function are not homogeneities (the *p*-values are less than significance level of 0.05 for each function). Thus, it is concluded that there is a difference between the variances of the distributions of the four algorithms for each of eight functions. Since the conditions of normality (for most functions) and homoscedasticity (for all functions) are not verified, the parametric tests are not appropriate for further analysis. Therefore, in this paper, the non-parametric tests are applied for single-function analysis. First, the Wilcoxon test for pairwise comparisons is applied. The Wilcoxon test is the analysis of significance of the difference between the samples of results' mean ranks of two algorithms. Table 5 presents the *p*-values obtained by non-parametric Wilcoxon test and parametric paired *t*-test (which is used for comparisons of obtained *p*-values). Parametric paired *t*-test determines whether the mean difference between two samples of results is zero. Also, the differences of average error rates of pairs of the algorithms are obtained and, for functions f6–f8, are given in Table 5. It is obvious that *p*-values obtained by Wilcoxon test and paired *t*-test are similar. However, in two cases (in function f3), the *p*-values are quite different. In these two cases, the Wilcoxon non-parametric test is taken as acceptable because the normality condition is not verified in the results of function f3 (Table 4). The *p*-values obtained by Wilcoxon test are less than significance level of 0.05 in 22 cases. It means there is a difference between the performance of compared algorithms and algorithm that has a smaller average error rate has a better performance. Also, if the difference of average error rates is positive, the best performed algorithm is second in the pair, and vice verse if the difference is negative. In two cases, the *p*-values are greater than significance value, which means that there is no difference between the algorithms' behavior in these cases. Given the *p*-values and average error rate values, the pairwise comparisons show that the

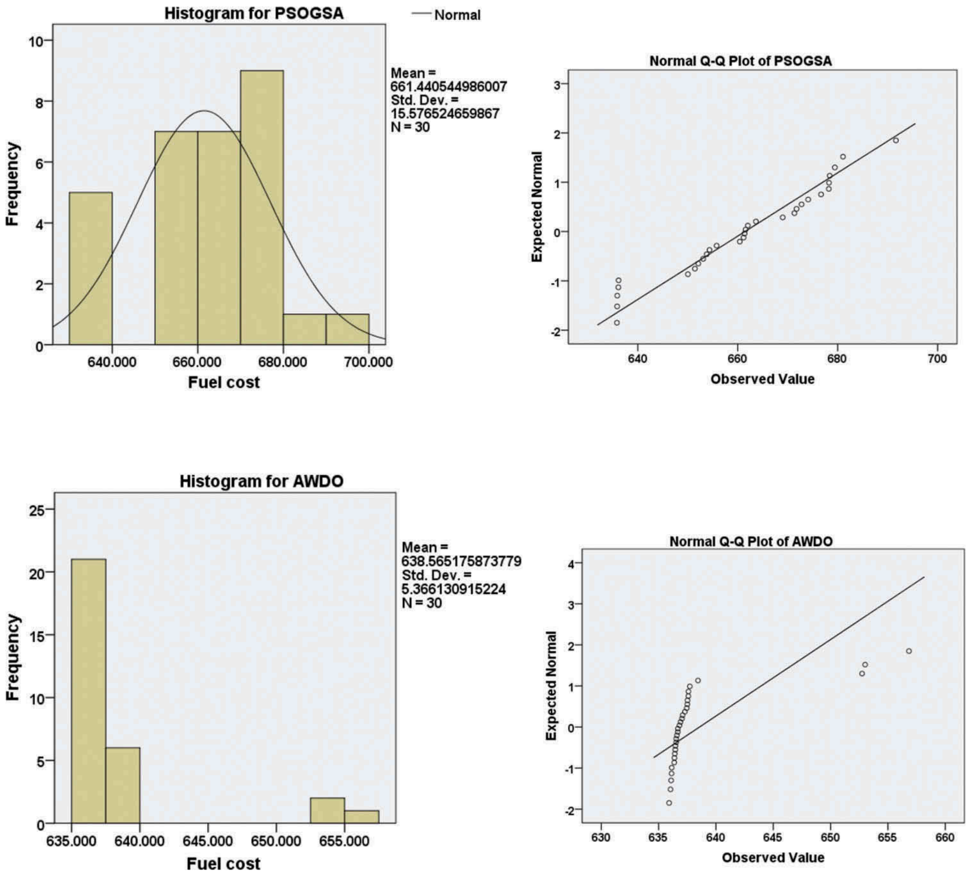

Figure 1. The histograms and Q–Q plots of PSOGSA and AWDO over function f7.

PSOGSA is the best in minimizing the functions f1 and f2, the AWDO is the best for the functions f3, f6 and f8, the MSA is the best for the function f4 and the FA is the best for the functions f5 and f7. The true value of p for combining pairwise comparisons of one algorithm with the rest of them (Garcia et al. 2009; Kaveh 2014) is calculated as follows:

$$p = 1 - \prod_{i=1}^{k} (1 - p_i), \tag{4}$$

where p_i is the probability value of ith pairwise comparison and k is the number of comparisons of one algorithm. Table 6 gives the true values of p for algorithms with better performance, for all functions. If consider that obtained p in (4) is significance level α, from these values of α, the corresponding values of the confidence intervals are calculated as 100 (1-α) and shown in Table 6. From Tables 6 and 1 can be concluded:

Table 5. *p*-values for paired *t*-test and Wilcoxon test and difference of error rates in the single-problem analysis (the functions f6–f8).

Function	Algorithms pair	*t*-test	Wilcoxon test	Difference of error rates
f6	PSOGSA-AWDO	0.000	0.000026	3.597774E+ 00
	PSOGSA-MSA	0.000	0.000332	2.675756E+ 00
	PSOGSA-FA	0.000	0.000041	3.218640E+ 00
	AWDO-MSA	0.007	0.005320	−9.220173E-01
	AWDO-FA	0.186	0.328571	−3.791337E-01
	MSA-FA	0.110	0.059836	5.428836E-01
f7	PSOGSA-AWDO	0.000	0.000003	5.363813E+ 00
	PSOGSA-MSA	0.000	0.000003	5.368974E+ 00
	PSOGSA-FA	0.000	0.000003	5.368975E+ 00
	AWDO-MSA	0.011	0.093676	5.160390E-03
	AWDO-FA	0.011	0.035009	5.161720E-03
	MSA-FA	0.000	0.000002	1.329837E-06
f8	PSOGSA-AWDO	0.000	0.000012	2.710564E+ 00
	PSOGSA-MSA	0.000	0.000014	2.632804E+ 00
	PSOGSA-FA	0.000	0.000010	2.648642E+ 00
	AWDO-MSA	0.000	0.000010	−7.775964E-02
	AWDO-FA	0.046	0.036826	−6.192187E-02
	MSA-FA	0.669	0.054463	1.583777E-02

Table 6. True values of *p* and confidence interval for algorithms with better performance.

Function	Algorithm with better performance	True value of *p*	Confidence interval, %
f1	PSOGSA	0.000263	99.97
f2	PSOGSA	0.029752	97.02
f3	AWDO	0.040704	95.93
f4	MSA	0.008761	99.12
f5	FA	0.000010	100.00
f6	AWDO	0.005346	99.46
f7	FA	0.035014	96.50
f8	AWDO	0.036847	96.31

- The PSOGSA has the advantage over other algorithms in solving CEED problem with objective functions that do not take into account VPLE and losses in a system.
- The FA has the advantage in solving the CEED problem by taking into account system losses.
- The AWDO outperforms other algorithms in the case where the objective functions are the most complex, i.e. when both losses and VPLE are considered.

Multiple-Problem Analysis

For multiple-problem analysis, two or more algorithms are compared considering all functions simultaneously. For this comparison, the average values of the 30 runs of an algorithm in each function are obtained. These average values, in this paper, are mean values of error rates (average error rates) (Table 3). First, the tests of normality are applied to the multiple problem for

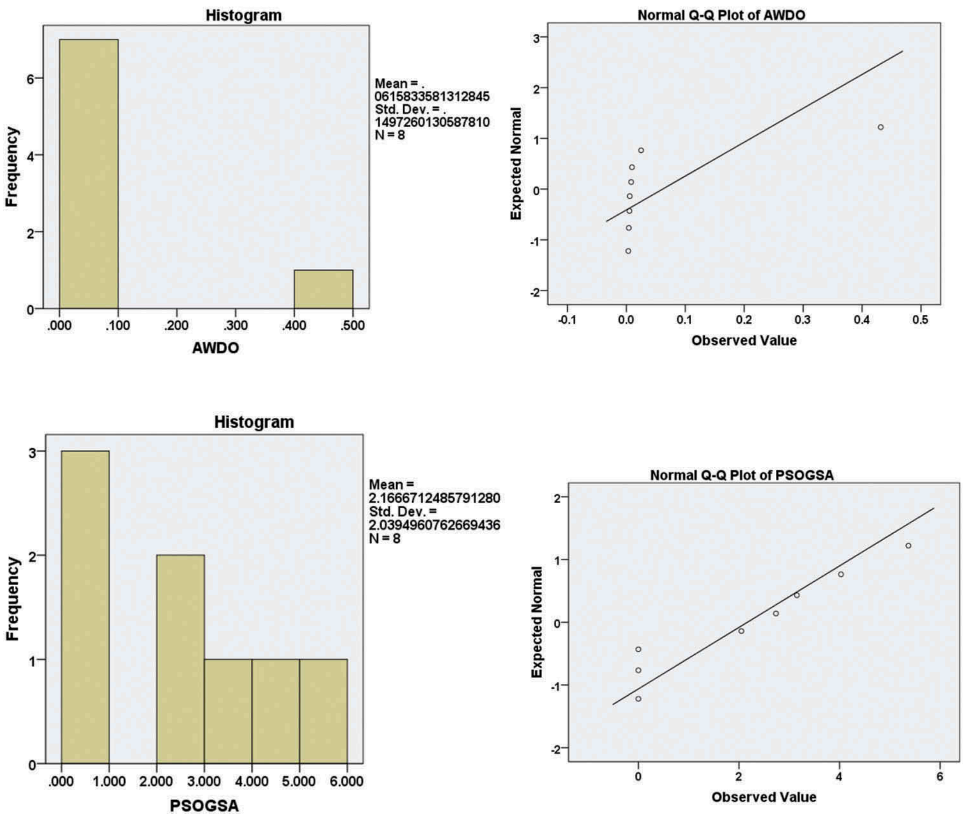

Figure 2. The histograms and Q–Q plots of AWDO and PSOGSA over all functions.

analysis of the conditions for the safe usage of parametric tests. Table 7 and Figure 2 show the results of applying the normality tests. As seen in Table 7, the normality condition is satisfied in the case of PSOGSA use but not satisfied in the case of other three algorithms use. The histograms and Q–Q plots (Figure 2), that are illustrated for PSOGSA and AWDO use, confirm the results presented in Table 7.

The result of Levene's test is $p = 0.00$ (the p-value is less than the significance level of 0.05) what shows that the sample variances of algorithms for all functions simultaneously are not homogeneities. Thus, it is concluded that there is a difference between the variances of the distributions of the four algorithms for all the eight functions simultaneously (for multiple problem).

Table 7. p-values for normality tests over multiple-problem analysis.

Algorithm	Kolmogorov-Smirnov	Shapiro-Wilk	D'Agostino-Pearson
PSOGSA	0.133	0.079	0.4306
AWDO	0.000	0.000	0.000
MSA	0.000	0.000	0.000
FA	0.000	0.000	0.000

Since the conditions of normality and homoscedasticity are not verified, the parametric tests are not appropriate for further analysis of this problem. Therefore, we apply the non-parametric tests for multiple problem analysis.

Non-Parametric Tests for Multiple-Problem Analysis

The Wilcoxon and Friedman tests are non-parametric tests that are applied in this paper for pairwise comparisons of algorithms in the multiple-problem analysis. In our case, both tests determine whether two sets of results obtained by means of two algorithms represent two populations with different median values. Table 8 presents the results obtained from Wilcoxon and Friedman tests: p-value for each pair of algorithms applied over all eight functions simultaneously. From Table 8, it can be seen that all values of p are $p > 0.05$ which means that compared algorithms behave the same, i.e. algorithms have the same performance and differences between the set of results obtained for each pair of algorithms are not statistically significant.

Conclusion

When solving complex optimization problems using MAs, often it is necessary to apply one MA to several functions separately or simultaneously. One such problem, for which a large number of MAs have been proposed in the recent literature, is the CEED problem. In this paper, a statistical analysis of the behavior of four MAs in solving eight functions describing the CEED problem was performed. For this analysis, parametric and non-parametric tests were applied. Two types of analysis are performed:

- analysis of the results obtained for each function independently.
- analysis of the results obtained for all functions simultaneously.

The tests of normality (Kolmogorov-Smirnov, Shapiro-Wilk and D'Agostino-Pearson) showed that the normality condition is satisfied in the case of PSOGSA use (for single functions and for all functions simultaneously) but not satisfied in the case of use of other three algorithms (FA, MSA and AWDO). The results of Levene's test showed that the conditions of

Table 8. p-values of Wilcoxon and Friedman tests over multiple-problem analysis.

Pairs of algorithms	Wilcoxon test	Friedman test
PSOGSA-AWDO	0.093	0.480
MSA-AWDO	0.674	0.157
FA-AWDO	0.674	0.157
MSA-PSOGSA	0.093	0.480
FA-PSOGSA	0.093	0.157
FA-MSA	0.889	0.157

homoscedasticity are not verified for results of single functions and for results of all functions too. Since the results of normality and homoscedasticity have not been met, it was been concluded that parametric tests are not appropriate for further analysis. Afterwards, the non-parametric Wilcoxon test and parametric paired *t*-test were applied over results of single functions and they showed that the results obtained using non-parametric and parametric tests in this case are similar. From this analysis, it follows that PSOGSA is the best in minimizing the functions f1 and f2, AWDO is the best for the functions f3, f6 and f8, MSA is the best for the function f4 and FA is the best for the functions f5 and f7. It means that one MA is not the best for each function of CEED problem. Therefore, different MAs should be used to solve various variants of the CEED problem. The analysis of results obtained for all functions simultaneously using the Wilcoxon's non-parametric test showed that the four compared MAs behave the same and MAs have the same performance, i.e. differences between the set of results obtained for each pair of algorithms are not statistically significant.

The framework which is given in this work can be applied for solving other multiple problems, with the aim of selecting the best optimization algorithm.

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