Journal of Advances in Mathematics and Computer Science

34(3): 1-16, 2019; Article no.JAMCS.52085

ISSN: 2456-9968

(Past name: British Journal of Mathematics & Computer Science, Past ISSN: 2231-0851)

A Stochastic Model with Jumps for Smoking

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

Article Information

DOI: 10.9734/JAMCS/2019/v34i3-430207

Editor(s):

(1) Dr. Feyzi Başar Professor, Department of Primary Mathematics Teacher Education, İnönü University, Turkey.

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(1) Francisco Bulnes, TESCHA, Mexico.

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Complete Peer review History: http://www.sdiarticle4.com/review-history/52085

Received: 09 August 2019

Accepted: 18 October 2019

Published: 01 November 2019

Original Research Article

Abstract

Some stochastic epidemiological models are less significant. They do not take into account some sudden events that could disrupt the behavior of the studied phenomenon. In this work, we introduce a white noise and jumps in a deterministic SIRS model for smoking to take into account of the effects of randomly fluctuation and such sudden factors respectively. First of all we prove that the solution of the stochastic differential equation with jumps of the new model is positive. Then we study the asymptotic behavior around the smoking-free equilibrium state and the smoking-present equilibrium state of the original deterministic model. Under certain conditions, we show that the solution oscillate respectively around these equilibrium states. We prove that the intensity of these oscillations depends on the magnitude of noise and the jump diffusion coefficient of our stochastic differential equation with jumps. To support our theoretical results, we realise numerical simulations. The observations confirm our conclusions.

Keywords: Smoking model; jump perturbation; global positive solution; asymptotic behavior; numerical simulation.

2010 Mathematics Subject Classification: 00A99; 97U60; 97U70.

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1 Introduction

C. Castillo-Garsow et al. established in [1] a model for smoking with three differential equations. The different unknowns P, S and Q_p represented respectively potential smokers (who don't smoke now), somkers and former smokers who have permanently quit smoking. This model was improved by Sharomi and Gumel in [2] by introducing a fourth class consists of people who give up smoking temporarily (Q_T). The total population is assumed to be constant, so P(t), S(t), $Q_T(t)$ and $Q_p(t)$ are, respectively, at time t, the proportions of the potential smokers, smokers, smokers who have quit smoking temporarily and smokers who have quit smoking permanently. This gives $P(t) + S(t) + Q_T(t) + Q_p(t) = 1$. The nonlinear equation describing the dynamics of smoking is:

$$\begin{cases} dP(t) = [\mu - \mu P(t) - \beta P(t)S(t)]dt \\ dS(t) = [-(\mu+\gamma)S(t) + \beta P(t)S(t) + \alpha Q_T(t)]dt \\ dQ_T(t) = [-(\mu+\alpha)Q_T(t) + \gamma(1-\sigma)S(t)]dt \\ dQ_p(t) = [-\mu Q_p(t) + \sigma\gamma S(t)]dt \end{cases} \tag{1.1}$$

α, β, μ, γ and σ are real constants on [0;1]. Their meanings are as follows.
μ is the non-smokers recruitment rate in the total population and also the death rate in each compartment.
β is the contact rate : it is the rate at which potentiel smokers become smokers by contact with smokers.
α is the rate at which smokers who temporarily quit smoking revert to smoking.
γ is the rate at which smokers left smoking and become smokers who have quit smoking permanently (Qp) or temporarily (Q_T).

σ is the rate of smokers who have left smoking to become smokers who have quit smoking permanently (Q_p). $1-\sigma$ is obviously the fraction of smokers who have left smoking to become smokers who have quit smoking temporarily (Q_T).

$P(t) + S(t) + Q_T(t) + Q_p(t) = 1$. So the study the system (1.1) can be reduced to that of the the following three-dimensional system (1.2) :

$$\begin{cases} dP(t) = [\mu - \mu P(t) - \beta P(t)S(t)]dt \\ dS(t) = [-(\mu+\gamma)S(t) + \beta P(t)S(t) + \alpha Q_T(t)]dt \\ dQ_T(t) = [-(\mu+\alpha)Q_T(t) + \gamma(1-\sigma)S(t)]dt \end{cases} \tag{1.2}$$

In [2], O. Shoromi et al. calculated R_s the basic reproduction number of the model (1.2).
$R_s = \dfrac{\beta(\mu+\alpha)}{\mu(\mu+\alpha)+\gamma(\sigma\alpha+\mu)}$. They call R_s the smokers generation number. It measures the average number of new smokers generated by a single smoker in a population of potential smokers. The model described by (1.2), admits two equilibria. When $R_s < 1$, we obtain the smoking-free equilibrium state $E_0 = (1,0,0)$ which is globally asymptotically stable ([2]). When $R_s > 1$, we obtain the smoking-present equilibrium state $E^* = (P^*, S^*, Q_T^*)$ where the components P^*, S^* and Q_T^* are calculated as $P^* = \dfrac{1}{R_s}$, $S^* = \dfrac{\mu}{\beta}(R_s - 1)$, $Q_T^* = \dfrac{\gamma(1-\sigma)}{\mu+\alpha}S^*$.

In [3], when $R_s > 1$, A. Lahrouz et al. proved that the smoking-present equilibrium state E^* of the system (1.2) is globally asymptotically stable in $\mathbb{R}^3_+$.

Some small randomly fluctuatations such as a decline in tobacco production, emigration or immigration may affect the model compartments. In this case the disturbances are modeled by white noise type which are proportional to P; S; Q_T.

The population may suffer sudden shocks which disrupt the habits. For example ; a very shocking medias campaigns against smoking by using fear in prevention could make a large group of people give up smoking. Such campaigns could also lead some people to never smoke.

Pictures and movies related to cancers caused by smoking are very dissuasive. The model described by the system (1.2) does not consider that circumstance. This is why we need to introduce jumps in this system to represent this sudden change.

Sudden high taxes on tobacco products are a proven way to reduce tobacco use especially among youth (see [4]).

Our aim is to study the asymptotic behavior of SIRS model for smoking with jumps symbolizing sudden changes as above.

In this work, from the model (1.2), we develop a model with jumps perturbation as in [5] :

$$\begin{cases} dP(t) =[\mu - \mu P(t) - \beta P(t)S(t)]dt + \sigma_1 P(t)dB_1(t) \\ \qquad + \int_Z C_1(z)P(t-)\tilde{N}(\mathrm{d}t, \mathrm{d}z) \\ dS(t) =[-(\mu+\gamma)S(t) + \beta P(t)S(t) + \alpha Q_T(t)]dt + \sigma_2 S(t)dB_2(t) \\ \qquad + \int_Z C_2(z)S(t-)\tilde{N}(\mathrm{d}t, \mathrm{d}z) \\ dQ_T(t) =[-(\mu+\alpha)Q_T(t) + \gamma(1-\sigma)S(t)]dt + \sigma_3 Q_T(t)dB_3(t) \\ \qquad + \int_Z C_3(z)Q_T(t-)\tilde{N}(\mathrm{d}t, \mathrm{d}z). \end{cases} \tag{1.3}$$

where $B_i, i = 1, 2, 3$ are independent Brownian motions defined on a stochastic basis $(\Omega, \mathcal{F}, (\mathcal{F}_{t\geq 0}), P)$ and $\sigma_i, i = 1, 2, 3$ are constants.

$X(t$-$)$ means the left limit of $X(t)$, $\tilde{N}(dt, dz)$ is a Poisson counting measure with the stationary compensator $\Pi(dz)dt$ and Π is defined on a measurable subset Z of $[0, \infty)$ with $\Pi(Z) < \infty$. For $i = 1, 2, 3, C_i > -1$.

For practical and realistic reasons, we consider that at the initial state, each compartment of our model is not empty.
As in similar work done in [6] and [7] our study is organized as follows. In Section 2, we prove that the equation (1.3) has a unique global and positive solution. In Section 3 et 4, we study respectively how the jumps influence the behavior around the equilibrium states E_0 and E^*. Section 5 presents some simulation with Matlab software by using numerical methods in [8] and [9].

2 Global Positive Solution

In this section, as in [5] and [10], Lyapunov methods are used to show that the solution of model (1.3) is positive and global.

For the jump diffusion coefficient we assume that for each $m > 0$ there exists $L_m > 0$ such that

- **(H1)** $\int_Z |H_i(x,z) - H_i(y,z)|^2 \Pi(\mathrm{d}z) \leq L_m|x-y|^2$, i=1, 2, 3, where $H_1(x,z) = C_1(z)P(x-)$, $H_2(x,z) = C_2(z)S(x-)$, $H_3(x,z) = C_3(z)Q_T(x-)$ with $|x| \vee |y| \leq m$
- **(H2)** $|\log(1 + C_i(z))| \leq K_1$, for $C_i(z) > -1$, where K_1 is positive constant, $i = 1, 2, 3$.

Let $\Delta = \{(x_1, x_2, x_3) \in \mathbb{R}_+^{*3} ; x_1 + x_2 + x_3 < 1\}$.

As in [10], we show that jump processes can suppress the explosion.

Theorem 2.1. *Assume* ***(H1)*** *and* ***(H2)*** *assumptions. Then for any given initial value* $(P(0), S(0), Q_T(0)) \in \Delta$*, the equation* (1.3) *has a unique global solution* $(P(t), S(t), Q_T(t)) \in \Delta$ *for all* $t \geq 0$ *almost surely.*

Proof. Using **(H1)** and that the dritft and difusion are locally Lipschitz, we deduce that for any given initial value $(P(0), S(0), Q_T(0)) \in \Delta$ there is a unique local solution (P, S, Q_T) on a random interval $[0, \tau_e[$ where τ_e is the explosion time.

To prove this local solution is global, we need to show that $\tau_e = \infty$ a.s. We willl also prove that for any given initial value $(P(0), S(0), Q_T(0)) \in \Delta$, the equation (1.3) has its solution $(P(t), S(t), Q_T(t)) \in \Delta$ for all $t \geq 0$ *almost surely.*

Let $m_0 > 0$ be sufficiently large so that $|P(0)|+|S(0)|+|Q_T(0)|$ lie within the interval $]\frac{1}{m_0}, \frac{m_0}{m_0+1}[$. We also have $P(0)$, $S(0)$ and $Q_T(0)$ in the interval $]\frac{1}{m_0}, m_0[$. For each integer $m \geq m_0$, define a new stopping time

$$\tau_m^{'} = \inf\{t \in [0, +\infty[: |P(t)| + |S(t)| + |Q_T(t)| \notin]\frac{1}{m}, \frac{m}{m+1}[\}.$$

Clearly, $\tau_m^{'}$ is increasing as $m \uparrow \infty$ a.s. Set $\tau_\infty^{'} = \lim_{m\to\infty} \tau_m^{'}$. If we can show that $\tau_\infty^{'} = \infty$ a.s is true, then $|P(t)| + |S(t)| + |Q_T(t)| < 1$ a.s.

For each integer $m \geq m_0$, define the stopping time

$$\tau_m = \inf\{t \in [0, \tau_e \wedge \tau_\infty^{'}[: P(t) \notin]\frac{1}{m}, m[\text{ or } S(t) \notin]\frac{1}{m}, m[\text{ or } Q_T(t) \notin]\frac{1}{m}, m[\}.$$

Clearly, τ_m is increasing as $m \uparrow \infty$ a.s. Set $\tau_\infty = \lim_{m\to\infty} \tau_m$, whence, $\tau_\infty \leq \tau_e$ and $\tau_\infty \leq \tau_\infty^{'}$ a.s. If we can show that $\tau_\infty = \infty$ a.s is true, then $\tau_e = \infty$ and $\tau_\infty^{'} = \infty$ a.s and $(P(t), S(t), Q_T(t)) \in \Delta$ a.s.

To prove that, we consider a C^2-function defined on $]0; +\infty[^3$ as following by
$F : (x, y, z) \mapsto (x - 1 - \log x) + (y - 1 - \log y) + (z - 1 - \log z)$.

Let $m \geq m_0$ and $T > 0$ be arbitrary. For $t \in [0; \tau_m \wedge T[$, $F(P(t), S(t), Q_T(t)) = (P(t) - 1 - \log P(t)) + (S(t) - 1 - \log S(t)) + (Q_T(t) - 1 - \log Q_T(t))$ is well defined.

Applying Itô´ s formula, we obtain

$$\begin{aligned} dF(P(t), S(t), Q_T(t)) &= LF(P(t), S(t), Q_T(t))dt \\ &+ \sigma_1(P(t) - 1)dB_1(t) \\ &+ \sigma_2(S(t) - 1)dB_2(t) \\ &+ \sigma_3(Q_T(t) - 1))dB_3(t) \\ &+ \int_Z [C_1(z)P(t-) - \log(1 + C_1(z)) \\ &+ C_2(z)S(t-) - \log(1 + C_2(z)) \\ &+ C_3(z)Q_T(t-) - \log(1 + C_3(z))]\tilde{N}(\mathrm{d}t, \mathrm{d}z) \end{aligned}$$

where

$$\begin{aligned}
LF(P(t),S(t),Q_T(t)) = {} & [4\mu+\alpha+\gamma+\beta P(t)+\beta S(t)+\gamma(1-\sigma)S(t) \\
& + \frac{1}{2}\sigma_1^2+\frac{1}{2}\sigma_2^2+\frac{1}{2}\sigma_3^2] \\
& - [\mu P(t)+(\mu+\gamma)S(t)+\mu Q_T(t)+\frac{\mu}{P(t)} \\
& + \alpha\frac{Q_T(t)}{S(t)}+\gamma(1-\sigma)\frac{S(t)}{Q_T(t)}] \\
& + \int_Z [C_1(z)-\log(1+C_1(z)) \\
& + C_2(z)-\log(1+C_2(z)) \\
& + C_3(z)-\log(1+C_3(z))]\,\mathrm{d}\Pi(z).
\end{aligned}$$

For $t \in [0;\tau_m \wedge T[$, $P(t), S(t), Q_T(t), \alpha, \beta, \gamma, \sigma$ and μ live in [0;1]. So we get

$$\begin{aligned}
& 4\mu+\alpha+\gamma+\beta P(t)+\beta S(t)+\gamma(1-\sigma)S(t) \; + \frac{1}{2}\sigma_1^2+\frac{1}{2}\sigma_2^2+\frac{1}{2}\sigma_3^2 \\
& \leq 4\mu+\alpha+\gamma+2\beta+\gamma(1-\sigma)+\frac{1}{2}(\sigma_1^2+\sigma_2^2+\sigma_3^2) = K_2
\end{aligned}$$

and

$$LF(P(t),S(t),Q_T(t)) \leq K_2+3K_3 \; = \; K$$

with

$$\begin{aligned}
K_3 = max\{ & \int_Z [C_1(z)-\log(1+C_1(z))]\,\mathrm{d}\Pi(z)\,, \\
& \int_Z [C_2(z)-\log(1+C_2(z))]\,\mathrm{d}\Pi(z)\,, \\
& \int_Z [C_3(z)-\log(1+C_3(z))]\,\mathrm{d}\Pi(z)\}.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\int_0^{\tau_m\wedge T} \mathrm{d}F(P(t),S(t),Q_T(t)) \leq {} & \int_0^{\tau_m\wedge T} K\,\mathrm{d}t \\
& + \int_0^{\tau_m\wedge T} \sigma_1(P(t)-1)(P(t)-P^*)dB_1(t) \\
& + \int_0^{\tau_m\wedge T} \sigma_2(S(t)-1)(S(t)-S^*)dB_2(t) \\
& + \int_0^{\tau_m\wedge T} \sigma_3(Q_T(t)-1)(Q_T(t)-Q_T^*)dB_3(t) \\
& + \int_0^{\tau_m\wedge T}\int_Z [C_1(z)P(t-)-\log(1+C_1(z)) \\
& + C_2(z)S(t-)-\log(1+C_2(z)) \\
& + C_3(z)Q_T(t-)-\log(1+C_3(z))]\tilde{N}(\,\mathrm{d}t,\mathrm{d}z).
\end{aligned}$$

Taking expectation, yields

$$\mathbb{E}(F(P(\tau_m \wedge T), S(\tau_m \wedge T), Q_T(\tau_m \wedge T))) \leq F(P(0), S(0), Q_T(0)) + KT = K' \qquad (2.1)$$

Define for each $u > 1$,

$\mu(u) = inf\left\{F(x) : \ x = (x_1, x_2, x_3) \in]0; +\infty[^3 \ and \, x_i \geq u \ or \ x_i \leq \frac{1}{u} \ for \ some \ i = 1, 2, 3\right\}$. Due to the property of the function $h(x) := x - 1 - lnx$, $x > 0$, we see that $\lim_{x\to+\infty} h(x) = +\infty$ and $\lim_{\substack{x\to 0\\ x>0}} h(x) = +\infty$
and hence
$\lim_{u\to+\infty} \mu(u) = +\infty$.

We know that if X is a Levy process then, for each $t \geq 0$ we have $\Delta X(t) = 0$ almost surely where $\Delta X(t) = X(t) - X(t^-)$.

P, S, Q_T are Levy processes. So we obtain almost surely from (2.1) :

$$\begin{aligned}\mu(m)\mathbb{P}(\tau_m \leq T) &\leq \mathbb{E}(F(P(\tau_m \wedge T), S(\tau_m \wedge T), Q_T(\tau_m \wedge T))\mathbb{I}_{\{\tau_m \leq T\}}) \\ &\leq \mathbb{E}(F(P(\tau_m \wedge T), S(\tau_m \wedge T), Q_T(\tau_m \wedge T))) \\ &\leq K'.\end{aligned}$$

We deduce

$$\mathbb{P}(\tau_m \leq T) \leq \frac{K'}{\mu(m)}.$$

Letting $m \longrightarrow +\infty$ yields

$\mathbb{P}(\tau_\infty \leq T) = 0$.

Since T is arbitrary, we must have

$\mathbb{P}(\tau_\infty = +\infty) = 1$ a.s.

So $\tau_e = +\infty$ a.s. and the equation (1.3) has a unique global positive solution $X(t)$ in Δ for $t \geq 0$. □

3 Asymptotic Behavior around the Smoking-free Equilibrium State of the Deterministic Model

Considering the system (1.2), if $R_s = \dfrac{\beta(\mu+\alpha)}{\mu(\mu+\alpha)+\gamma(\sigma\alpha+\mu)} < 1$, the solution $E_0 = (1, 0, 0)$ is globally asymptotically stable. For now, we are not be able to explain the solution of the system (1.3). So when $R_s < 1$, we will study the asymptotic behavior of our stochastic model with jumps around E_0.

Theorem 3.1. *Let* $(P(t), S(t), Q_T(t))$, *be the solution of the system* (1.3) *with initial value* $(P(0), S(0), Q_T(0)) \in \Delta$. *If* $R_s = \dfrac{\beta(\mu+\alpha)}{\mu(\mu+\alpha)+\gamma(\sigma\alpha+\mu)} < 1$ *and* $\beta < \mu + \gamma$, *then there exists* $K > 0$ *such that*

$$\limsup_{t\to\infty} \frac{1}{t} E\int_0^t [(P(\tau)-1)^2 + S^2(\tau) + Q_T(\tau)]\,\mathrm{d}\tau \leq \frac{\sigma_1^2+\sigma_2^2+\int_Z (C_1^2(z)+C_2^2(z))|\Pi(\,\mathrm{d}z)}{K'}$$

where

$$K' = min\{\mu\,;\ \frac{K}{2}[[\mu(\mu+\alpha)+\gamma(\mu+\sigma\alpha)](1-R_s) - \frac{2\alpha}{K}]\}.$$

Proof. First, change the variables $x = P - 1$, $y = S$, $w = Q_T$, then system (1.3) can be written as

$$\begin{cases} dx(t) = [\mu - \mu(x(t)+1) - \beta(x(t)+1)y(t)]dt + \sigma_1(x(t)+1)dB_1(t) \\ \qquad + \int_Z C_1(z)(x(t-)+1)\tilde{N}(\,\mathrm{d}t, \mathrm{d}z) \\ dy(t) = [-(\mu+\gamma)y(t) + \beta(x(t)+1)y(t) + \alpha w(t)]dt + \sigma_2 y(t)dB_2(t) \\ \qquad + \int_Z C_2(z)y(t-)\tilde{N}(\,\mathrm{d}t, \mathrm{d}z) \\ dw(t) = [-(\mu+\alpha)w(t) + \gamma(1-\sigma)y(t)]dt + \sigma_3 w(t)dB_3(t) \\ \qquad + \int_Z C_3(z)w(t-)\tilde{N}(\,\mathrm{d}t, \mathrm{d}z). \end{cases} \tag{3.1}$$

We deduce that $-1 \leq x \leq 0$, $0 \leq y \leq 1$ and $0 \leq w \leq 1$.

Define a C^2-function by

$F(x, y, w) = (x + y)^2 + e_1 y + e_2 w$

where e_1 and e_2 are positive constants to be determined later. Then the function F is positive definite, and

$$\begin{aligned} dF(x(t), y(t), w(t)) = & LFdt \\ & + 2\sigma_1(x(t)+y(t))(x(t)+1)dB_1(t) \\ & + \sigma_2[2(x(t)+y(t)) + e_1]y(t)dB_2(t) + \sigma_3 e_2 w(t)dB_3(t) \\ & + \int_Z \{[(C_1(z)+1)^2 - 1]x^2(t-) + [(C_2(z)+1)^2 - 1]y^2(t-) \\ & + 2[(C_1(z)+1)(C_2(z)+1) - 1]x(t-)y(t-) + 2C_1(z)C_2(z)x(t-) \\ & + [2C_1(z)(C_2(z)+1) + e_1 C_2(z)]y(t-) \\ & + e_2 C_3(z)w(t-)\}\tilde{N}(\,\mathrm{d}t, \mathrm{d}z) \end{aligned} \tag{3.2}$$

where

$$\begin{aligned} LF = & -2\mu x^2 + (-4\mu - 2\gamma + \beta e_1)xy + [-(\mu+\gamma-\beta)e_1 + \gamma(1-\sigma)e_2]y + 2\alpha xw \\ & + [2\alpha y + \alpha e_1 - (\mu+\alpha)e_2]w - 2(\mu+\gamma)y^2 + [\sigma_1(x+1) + \sigma_2 y]^2 \\ & + \int_Z [(x+1)C_1(z) + yC_2(z)]^2 \Pi(\,\mathrm{d}z). \end{aligned}$$

Using $-1 \leq x \leq 0$, $0 \leq y \leq 1$, $0 \leq w \leq 1$, the fact α , β , γ , σ, μ are positves in $]0; 1[$

(implying that $2\alpha xw \leq 0$) and the inequality $(a+b)^2 \leq 2(a^2+b^2)$, we get

$$\begin{aligned} LF \leq & -2\mu x^2 + (-4\mu - 2\gamma + \beta e_1)xy + [-(\mu+\gamma-\beta)e_1 + \gamma(1-\sigma)e_2]y \\ & + [2\alpha + \alpha e_1 - (\mu+\alpha)e_2]w - 2(\mu+\gamma)y^2 + 2(\sigma_1^2+\sigma_2^2) \\ & + 2\int_Z (C_1^2(z) + C_2^2(z))\Pi(\,\mathrm{d}z). \end{aligned}$$

We assume that $\beta < \mu + \gamma$ so that $\mu + \gamma - \beta > 0$.

We choose $e_1 = K\gamma(1-\sigma)$ and $e_2 = K(\mu+\gamma-\beta)$ with $K > 0$ as

$K\beta\gamma(1-\sigma) > 4\mu + 2\gamma$ and $\dfrac{2\alpha}{K} < [\mu(\mu+\alpha) + \gamma(\sigma\alpha+\mu)](1-R_s)$.

In these cases the following is deduced :

$-4\mu - 2\gamma + \beta e_1)xy \leq 0$, $-(\mu+\gamma-\beta)e_1 + \gamma(1-\sigma)e_2 = 0$ and

$$\begin{aligned} & 2\alpha + \alpha e_1 - (\mu+\alpha)e_2 \\ & = 2\alpha + \alpha K\gamma(1-\sigma) - (\mu+\alpha)K(\mu+\gamma-\beta) \\ & = K[\frac{2\alpha}{K} + +\alpha\gamma(1-\sigma) - (\mu+\gamma-\beta)(\mu+\alpha)] \\ & = K[\frac{2\alpha}{K} + \beta(\mu+\alpha) - \mu(\mu+\alpha) - \gamma(\mu+\alpha) + \alpha\gamma - \alpha\gamma\sigma] \\ & = K[\frac{2\alpha}{K} + \beta(\mu+\alpha) - [\mu(\mu+\alpha) + \gamma(\mu+\sigma\alpha)]] \\ & = K[\frac{2\alpha}{K} + [\mu(\mu+\alpha) + \gamma(\mu+\sigma\alpha)](\frac{\beta(\mu+\alpha)}{\mu(\mu+\alpha)+\gamma(\mu+\sigma\alpha)} - 1)] \\ & = K[\frac{2\alpha}{K} + [\mu(\mu+\alpha) + \gamma(\mu+\sigma\alpha)](R_s - 1)] < 0. \end{aligned}$$

Finally we get

$$\begin{aligned} LF \leq & -2\mu x^2 - 2(\mu+\gamma)y^2 - K[[\mu(\mu+\alpha) + \gamma(\mu+\sigma\alpha)](1-R_s) - \frac{2\alpha}{K}]w \\ & + 2(\sigma_1^2+\sigma_2^2) + 2\int_Z (C_1^2(z) + C_2^2(z))\Pi(\,\mathrm{d}z). \end{aligned} \tag{3.3}$$

Integrating both sides of (3.2) from 0 to t, then taking expectation, yields

$$0 \leq EF(x(t),y(t),w(t)) = F(x(0),y(0),w(0)) + E\int_0^t LF(x(\tau)+y(\tau)+w(\tau)]\,\mathrm{d}\tau$$

By (3.3) we have

$$\begin{aligned} & \mathbb{E}[\int_0^t \{2\mu x^2(\tau) + 2(\mu+\gamma)y^2(\tau) + K[[\mu(\mu+\alpha) + \gamma(\mu+\sigma\alpha)](1-R_s) - \frac{2\alpha}{K}]w^2(\tau)\}\,\mathrm{d}\tau] \\ & \leq F(x(0),y(0),w(0)) + 2[\sigma_1^2 + 2\sigma_2^2\int_Z (C_1^2(z) + C_2^2(z))\Pi(\,\mathrm{d}z]. \end{aligned}$$

Therefore

$$\limsup_{t\to\infty} \tfrac{1}{t}E\int_0^t [(P(\tau)-1)^2 + S^2(\tau) + Q_T(\tau)]\,\mathrm{d}\tau \leq \frac{\sigma_1^2 + \sigma_2^2 + \int_Z (C_1^2(z) + C_2^2(z))|\Pi(\,\mathrm{d}z)}{K'}$$

where

$K' = min\{\mu\,;\,\mu+\gamma\,;\,\frac{K}{2}[[\mu(\mu+\alpha)+\gamma(\mu+\sigma\alpha)](1-R_s)-\frac{2\alpha}{K}]\} = min\{\mu\,;\,\frac{K}{2}[[\mu(\mu+\alpha)+\gamma(\mu+\sigma\alpha)](1-R_s)-\frac{2\alpha}{K}]\}$ □

Remark 3.1. We have just proved that, when $R_s < 1$, with a condition on some constants of the system, the solution of (1.2) oscillates more closely around the smoking-free equilibrium state as the intensity of the noise and the jumps decreases.

4 Asymptotic Behavior around the Smoking-present Equilibrium State of the Deterministic Model

We are not able to explain the solution of the sytem system (1.3). So, in the case where $R_s > 1$, we want to know if the solution system (1.3) can oscillate around the smoking-present equilibrium of the model (1.2).

Theorem 4.1. *Let* $(P(t), S(t), Q_T(t))$, *be the solution of the system* (1.3) *with initial value* $(P(0), S(0), Q_T(0)) \in \Delta$. *If* $R_s = \frac{\beta(\mu+\alpha)}{\mu(\mu+\alpha)+\gamma(\sigma\alpha+\mu)} > 1$ *and the condition* $\beta < \mu+\gamma+\frac{\mu(\mu+\gamma\sigma)(R_s-1)}{2\mu+\gamma\sigma}$ *is satisfied,*

then

$$\limsup_{t\to\infty}\frac{1}{t}E\int_0^t [(P(\tau)+Q_T(\tau)-P^*-Q_T^*)^2+(S(\tau)-S^*)^2+(Q_T(\tau)-Q_T^*)^2]\,\mathrm{d}\tau \leq \frac{M}{\mu}$$

where

$$M = \frac{1}{2}[3(\sigma_1^2+\sigma_2^2+\sigma_3^2)+m\sigma_2^2+v\sigma_3^2]+\int_Z \{m|C_2(z)|+v|C_3(z)|+\frac{3}{2}C_1^2(z)+2C_2^2(z)+2C_3^2(z)\}\Pi(\,\mathrm{d}z)$$

with $m = \frac{2\mu+\gamma\sigma}{\mu(R_s-1)}$ *and* $v = \frac{2\mu+\gamma\sigma}{\gamma(1-\sigma)}$.

Proof. Define $F : (x, y, z) \mapsto F_1(x, y, z) + F_2(x, y, z)$,
where $F_1 : (x, y, z) \mapsto \frac{1}{2}(x-P^*+y-S^*+z-Q_T^*)^2$, $F_2 : (x, y, z) \mapsto \frac{1}{2}m(y-S^*)^2+\frac{1}{2}v(z-Q_T^*)^2$,

where m and v are positive constants to be determined later.

F is positive definite.

By using of Itô´ s formula, we obtain

$$\begin{aligned}
dF_1(P(t), S(t), Q_T(t)) =& LF_1(t)dt + (P(t)-P^*+S(t)-S^*+Q_T(t)-Q_T^*)\times \\
& [\sigma_1 P(t)dB_1(t)+\sigma_2 S(t)dB_2(t)+\sigma_3 Q_T(t)dB_3(t)] \\
& + \int_Z \{[P(t)-P^*+S(t)-S^*+Q_T(t)-Q_T^*] \\
& \times [C_1(z)P(t-)+C_2(z)S(t-)+C_3(z)Q_T(t-)] \\
& + \frac{1}{2}[C_1(z)P(t-)+C_2(z)S(t-)+C_3(z)Q_T(t-)]^2\}\tilde{N}(\,\mathrm{d}t, \mathrm{d}z)
\end{aligned}$$

with

$$\begin{aligned}LF_1(t) =& -\left(P(t)-P^*+S(t)-S^*+Q_T(t)-Q_T^*\right)\\&\times\left[\mu(P(t)-P^*)+\mu(Q_T(t)-Q_T^*)+(\mu+\gamma\sigma)(S(t)-S^*)\right]\\&+\frac{1}{2}[\sigma_1 P(t)+\sigma_2 S(t)+\sigma_3 Q_T(t)]^2\\&+\frac{1}{2}\int_Z [C_1(z)P(t-)+C_2(z)S(t-)+C_3(z)Q_T(t-)]^2\Pi(\,dz)\end{aligned}$$

By using $\mu=\mu P^*+\beta P^*S^*$, $-(\mu+\gamma)S^*+\beta P^*S^*+\alpha Q_T^*=0$ and $-(\mu+\alpha)Q_T^*+\gamma(1-\sigma)S^*=0$ (these egalities from the fact that (P^*,S^*,Q_T^*) is the smoking-present equilibrium of the model (1.2)), we obtain

$$\begin{aligned}LF_1(t) =& -\mu(P(t)+Q_T(t)-P^*-Q_T^*)^2-(\mu+\gamma\sigma)(S(t)-S^*)^2\\&-(2\mu+\gamma\sigma)(P(t)-P^*)(S(t)-S^*)-(2\mu+\gamma\sigma)(Q_T(t)-Q_T^*)(S(t)-S^*)\\&+\frac{1}{2}[\sigma_1 P(t)+\sigma_2 S(t)+\sigma_3 Q_T(t)]^2\\&+\frac{1}{2}\int_Z [C_1(z)P(t-)+C_2(z)S(t-)+C_3(z)Q_T(t-)]^2\Pi(\,dz)\end{aligned}$$

In view of the elementary inegality $(a+b+c)^2\le 3(a^2+b^2+c^2)$ and since P , S , Q_T live in [0;1], we get

$$\begin{aligned}LF_1(t) \le& -\mu(P(t)+Q_T(t)-P^*-Q_T^*)^2-(\mu+\gamma\sigma)(S(t)-S^*)^2\\&-(2\mu+\gamma\sigma)(P(t)-P^*)(S(t)-S^*)-(2\mu+\gamma\sigma)(Q_T(t)-Q_T^*)(S(t)-S^*)\\&+\frac{3}{2}(\sigma_1^2+\sigma_2^2+\sigma_3^2)\\&+\frac{3}{2}\int_Z [C_1^2(z)+C_2^2(z)+C_3^2(z)]\Pi(\,dz).\end{aligned}\tag{4.1}$$

We also have

$$\begin{aligned}dF_2(P(t),S(t),Q_T(t))) =& LF_2(t)dt+\sigma_2 mS(t)(S(t)-S^*)dB_2(t)\\&+\sigma_3 vQ_T(t)(Q_T(t)-Q_T^*)dB_3(t)\\&+\int_Z\{\frac{1}{2}m(S(t-)-S^*+C_2(z)S(t-))^2\\&+\frac{1}{2}v(Q_T(t-)-Q_T^*+C_3(z)Q_T(t-))^2\\&-\frac{1}{2}m(S-S^*)-\frac{1}{2}v(Q_T-Q_T^*)\}\tilde{N}(\,dt,\mathrm{d}z).\end{aligned}$$

with

$$\begin{aligned}LF_2(t) =& m(S(t)-S^*)\\&\times[-(\mu+\gamma)(S(t)-S^*)+\beta(P(t)S(t)-P^*S^*)+\alpha(Q_T(t)-Q_T^*)]\\&+v(Q_T(t)-Q_T^*)[-(\mu+\alpha)(Q_T(t)-Q_T^*)+\gamma(1-\sigma)(S(t)-S^*)]\\&+\frac{1}{2}[m\sigma_2^2S^2(t)(S(t)-S^*)+v\sigma_3^2Q_T^2(t)(Q_T(t)-Q_T^*)]\\&+\int_Z\{m[C_2(z)S(t-))(S(t-)-S^*)+\frac{1}{2}C_2^2(z)S^2(t-))]\\&+v[C_3(z)Q_T(t-)(Q_T(t-)-Q_T^*)+\frac{1}{2}C_3^2(z)Q_T^2(t-)]\}\Pi(\,dz).\end{aligned}$$

Since $PS - P^*S^* = S^*(P - P^*) + P(S - S^*)$ we have,

$$\begin{aligned}LF_2(t) = &-(\mu+\gamma)m(S(t)-S^*)^2 - (\mu+\alpha)v(Q_T(t)-Q_T^*)^2\\&+ m\beta(S(t)-S^*)[S^*(P(t)-P^*)+P(t)(S(t)-S^*)]\\&+\gamma(1-\sigma)v(S(t)-S^*)(Q_T(t)-Q_T^*)\\&+\frac{1}{2}[m\sigma_2^2S^2(t)(S(t)-S^*)+v\sigma_3^2Q_T^2(t)(Q_T(t)-Q_T^*)]\\&+\int_Z\{m[C_2(z)S(t-))(S(t-)-S^*)+\frac{1}{2}C_2^2(z)S^2(t-))]\\&+v[C_3(z)Q_T(t-)(Q_T(t-)-Q_T^*)+\frac{1}{2}C_3^2(z)Q_T^2(t-)]\}\Pi(\,\mathrm{d}z)\end{aligned}$$

$$\begin{aligned}= &-(\mu+\gamma-\beta P(t))m(S(t)-S^*)^2-(\mu+\alpha)v(Q_T(t)-Q_T^*)^2\\&+m\beta S^*(S(t)-S^*)(P(t)-P^*)+\gamma(1-\sigma)v(S(t)-S^*)(Q_T(t)-Q_T^*)\\&+\frac{1}{2}[m\sigma_2^2S^2(t)(S(t)-S^*)+v\sigma_3^2Q_T^2(t)(Q_T(t)-Q_T^*)]\\&+\int_Z\{m[C_2(z)S(t-))(S(t-)-S^*)+\frac{1}{2}C_2^2(z)S^2(t-))]\\&+v[C_3(z)Q_T(t-)(Q_T(t-)-Q_T^*)+\frac{1}{2}C_3^2(z)Q_T^2(t-)]\}\Pi(\,\mathrm{d}z).\end{aligned}$$

Since P , S , Q_T , P^* , S^* , Q_T^*,$P-P^*$, $S-S^*$, $Q_T-Q_T^*$ live in [-1;1] we get

$$\begin{aligned}LF_2(t) \le &-(\mu+\gamma-\beta)m(S(t)-S^*)^2-(\mu+\alpha)v(Q_T(t)-Q_T^*)^2\\&+m\beta S^*(S(t)-S^*)(P(t)-P^*)+\gamma(1-\sigma)v(S(t)-S^*)(Q_T(t)-Q_T^*)\\&+\frac{1}{2}(m\sigma_2^2+v\sigma_3^2)\\&+\int_Z\{m[C_2(z)S(t-))(S(t-)-S^*)+\frac{1}{2}C_2^2(z)]\\&+v[C_3(z)Q_T(t-)(Q_T(t-)-Q_T^*)+\frac{1}{2}C_3^2(z)]\}\Pi(\,\mathrm{d}z).\end{aligned}\tag{4.2}$$

By using (4.1) and (4.2), we get

$$\begin{aligned}LF(t) \le &-\mu(P(t)+Q_T(t)-P^*-Q_T^*)^2\\&-[\mu+\gamma\sigma+m(\mu+\gamma-\beta)](S(t)-S^*)^2-(\mu+\alpha)v(Q_T(t)-Q_T^*)^2\\&+[m\beta S^*-(2\mu+\gamma\sigma)](S(t)-S^*)(P(t)-P^*)\\&+[\gamma(1-\sigma)v-(2\mu+\gamma\sigma)](S(t)-S^*)(Q_T(t)-Q_T^*)\\&+\frac{1}{2}[3(\sigma_1^2+\sigma_2^2+\sigma_3^2)+m\sigma_2^2+v\sigma_3^2]\\&+\int_Z\{m|C_2(z)|+v|C_3(z)|+\frac{3}{2}C_1^2(z)+2C_2^2(z)+2C_3^2(z)\}\Pi(\,\mathrm{d}z).\end{aligned}$$

Now we choose $m = \dfrac{2\mu+\gamma\sigma}{\mu(R_s-1)}$ and $v = \dfrac{2\mu+\gamma\sigma}{\gamma(1-\sigma)}$ such that $m\beta S^* - (2\mu+\gamma\sigma) = 0$ and $\gamma(1-\sigma)v - (2\mu+\gamma\sigma) = 0$. We also assume $\beta < \mu+\gamma+\dfrac{\mu+\gamma\sigma}{m}$ i.e $\beta < \mu+\gamma+\dfrac{\mu(\mu+\gamma\sigma)(R_s-1)}{2\mu+\gamma\sigma}$ so that $\mu+\gamma\sigma+m(\mu+\gamma-\beta) > 0$.
Finally, we get

$$\begin{aligned}LF(t) \le &-\mu(P(t)+Q_T(t)-P^*-Q_T^*)^2\\&-[\mu+\gamma\sigma+m(\mu+\gamma-\beta)](S(t)-S^*)^2-(\mu+\alpha)v(Q_T(t)-Q_T^*)^2+M\end{aligned}\tag{4.3}$$

where $M = \frac{1}{2}[3(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) + m\sigma_2^2 + v\sigma_3^2] + \int_Z \{m|C_2(z)| + v|C_3(z)| + \frac{3}{2}C_1^2(z) + 2C_2^2(z) + 2C_3^2(z)\}\Pi(\,\mathrm{d}z)$.
Therefore integrating both sides of $dF = dF_1 + dF_2$ from 0 to t, then taking expectations, yields

$$0 \leq EF(X(t)) = F(X(0)) + E\int_0^t LF(X(\tau))\,\mathrm{d}\tau.$$

Considering inequality (4.3) and letting $t \to \infty$ we have

$$\limsup_{t\to\infty} \frac{1}{t}E\int_0^t [(P(\tau) + Q_T(\tau) - P^* - Q_T^*)^2 + (S(\tau) - S^*)^2 + (Q_T(\tau) - Q_T^*)^2]\,\mathrm{d}\tau \leq \frac{M}{\mu}$$

because $\mu = min\{\mu\,,\, \mu + \gamma\sigma + m(\mu + \gamma - \beta)\,,\, (\mu + \alpha)v\}$. □

Remark 4.1. With certain conditions, we obtain that the solution of model (1.3) fluctuates more and more closely around the smoking-present equilibrium state of the deterministic model.

5 Numerical Simulations

Using Matlab software, we illustrate now some sample paths of solution of the model (1.3) for different values of the parameters. We observe that these paths are in agreement with theorical behavior.

We consider that the interference of Poisson jumps is negative for all compartments. This is motivated by conclusions of WHO (World Health Organization) report on the global tobacco epidemic in 2015 (see [4]).

On the other hand, as in Theorem 3.1 and Theorem 4.1, small intensities of jumps and white noise are conditions of oscillation of the solution around a equilibrium state. We want to observe it in the simulations. So these results motivated the choice of our jumps and noise coefficients in the simulations. We thus obtain simulations that do not involve large jumps. Indeed, in our model we integrate according to the random measure of compensated Poisson which corresponds to the sum of small jumps.

5.1 Oscillations around the Smoking-free Equilibrium State

For the first simulation, as in [5] and [9], we define $C_i(z) = k_i\frac{z^2}{1+z^2}, z \in [-1, 1], i = 1, 2, 3$ with $k_1 = -0.01$, $k_2 = -0.002$, $k_3 = -0.0025$ such that assumptions **(H1)** and **(H2)** of Section 2 are verified. We chose $\sigma_1 = 0.04$, $\sigma_2 = 0.03$, $\sigma_3 = 0.02$. We obtain the Figure 1.

For the second simulation, we use the following parameters: $\mu = 0.01$, $\gamma = 0.3$, $\alpha = 0.25$, and $\sigma = 0.4$, with $\beta = 0.1$. So $R_s = 0.73034$ and $\beta < \mu + \gamma$. Conditions in theorem 3.1 are satisfied. We get the Figure 2.

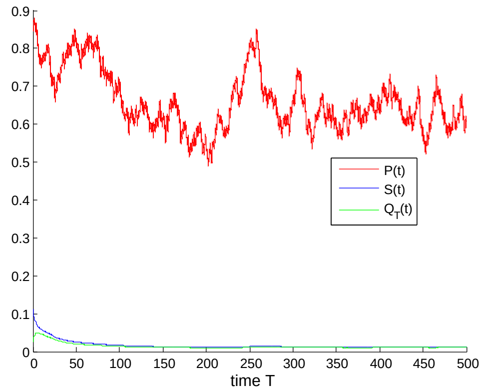

Fig. 1. Trajectory of the solution of the system (1.3) **for** $P(0) = 0.85955,\ S(0) = 0.111744,\ Q_T(0) = 0.025165,$ **,** $R_s < 1$ **and** $\beta < \mu + \gamma$**.**

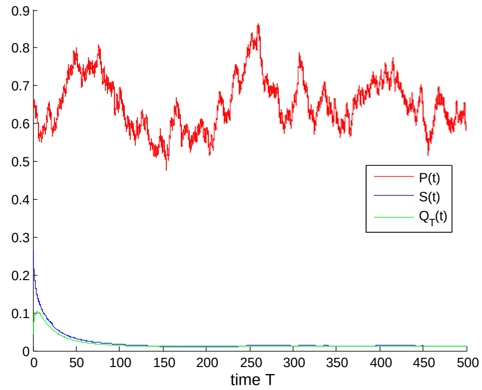

Fig. 2. Trajectory of the solution of the system (1.3) **for** $P(0) = 0.65000,\ S(0) = 0.26800,\ Q_T(0) = 0.04400$**,** $R_s < 1$ **and** $\beta < \mu + \gamma$.

5.2 Oscillations around the smoking-present equilibrium state

In this subsection, we reduce the parameter $M = \frac{1}{2}[3(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) + m\sigma_2^2 + v\sigma_3^2] + \int_Z \{m|C_2(z)| + v|C_3(z)| + \frac{3}{2}C_1^2(z) + 2C_2^2(z) + 2C_3^2(z)\}\Pi(\,dz)$ with $m = \frac{2\mu + \gamma\sigma}{\mu(R_s - 1)}$ and $v = \frac{2\mu + \gamma\sigma}{\gamma(1-\sigma)}$.
For that we choose $\mu = 0.01$, $\sigma_1 = 0.02$, $\sigma_2 = 0.005$, $\sigma_3 = 0.0055$, $k_1 = -0.01$, $k_2 = -0.002$, $k_3 = -0.0025$,

For the following simulation, we choose $\alpha = 0.25$, $\gamma = 0.3$, $\sigma = 0.4$ and the contact rate between potential smokers and smokers $\beta = 0.3$. So $R_s > 1$ and $\beta < \mu + \gamma + \frac{\mu(\mu + \gamma\sigma)(R_s - 1)}{2\mu + \gamma\sigma}$. Conditions of the theorem 4.1 are satisfied. We obtain the Figure 3. This figure shows the asymptotic behavior of the solution of the system (1.3). With our previous assumptions, this solution that oscillates around the smoking-free equilibrium state $(P^*; S^*; Q_T^*)$.

Next, we give an other numerical simulation to explain Theorem 4.1 by changing somme previous parameters. We take now : $\alpha = 0.35$, $\gamma = 0.3$, $\sigma = 0.2$ and $\beta = 0.3$. We obtain a greater value of R_s : $R_s = 3.913$. The condition $\beta < \mu + \gamma + \dfrac{\mu(\mu+\gamma\sigma)(R_s-1)}{2\mu+\gamma\sigma}$ is also verified. In this case, to better observe the asymptotic behavior of our model around the smoking-present equilibrium state of the deterministic model, we are forced to simulate the model on a larger time interval with more small jump diffusion coefficient. So we take the interval [0 ; 1000]. The figure Figure 4 correspond to these new hypotheses and support also our theory.

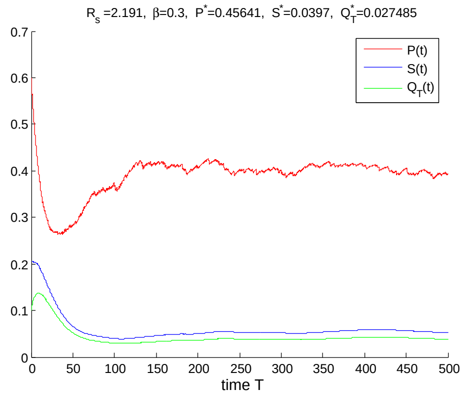

Fig. 3. Trajectory of the solution of the system (1.3) **for** $R_s > 1$ **,** $\beta < \mu + \gamma + \dfrac{\mu(\mu+\gamma\sigma)(R_s-1)}{2\mu+\gamma\sigma}$, $P(0) = 0.60000$, $S(0) = 0.20628$ **and** $Q_T(0) = 0.10000$.

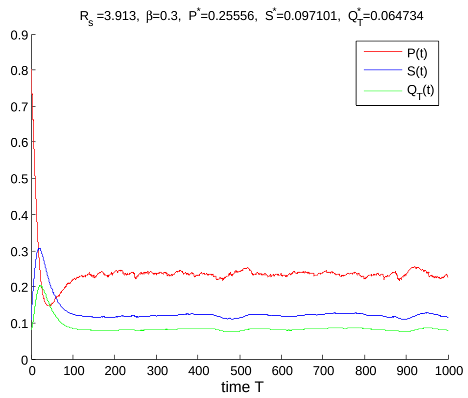

Fig. 4. Trajectory of the solution of the system (1.3) **for** $R_s > 1$ **,** $\beta < \mu + \gamma + \dfrac{\mu(\mu+\gamma\sigma)(R_s-1)}{2\mu+\gamma\sigma}$, $P(0) = 0.80301$, $S(0) = 0.10628$, **and** $Q_T(0) = 0.08260$.

6 Conclusion and Perspective

In this study, we modeled the state of smoking by a more realistic mathematical compartmental model. To do this, we have to improve a deterministic model that has a continuous structure. This continuity is not possible in the reality. In fact, the different compartments of the model represent classes of the population that are subject to non-continuous phenomena. We added to it some white noises that could symbolize small fluctuations due to population movements (immigration, emigration). The added jumps could model sudden phenomena like changes of attitudes due to a media campaign.
We have shown that by controlling some parameters of our model (the basic reproduction number and some constants of our system), we could approach a situation without smoking. In this case, the solution of our sytem oscillate more and more closely around the the smoking-free equilibrium state. Otherwise, smoking could be maintained at another appreciable stationary state by controlling certain constants. It is the situation where the solution of our sytem oscillate more and more closely around the the smoking-present equilibrium state.

The control of the constants of our sytem could be done by the aplication of some laws instituted by the public authorities. For example, a prohibition to smoke in a public area could lead to a reduction of the contact rate.

Acknowledgment

The corresponding author particularly thanks the second author who guided him and specially supervised him during all the works.

Competing Interests

The authors certify the absence of a competing interest.

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