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The Inverse Lomax-G Family with application to Breaking Strength Data

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Authors contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

We proposed a new class of distributions with two additional positive parameters called the Inverse Lomax-G (IL-G) class. A special case was discussed, by taking Weibull as a baseline. Different properties of the new family that hold for any type of baseline model are derived including moments, moment generating function, entropy for Renyi, entropy for Shanon, and order statistics. The performances of the maximum likelihood estimates of the parameters of the sub-model of the Inverse Lomax-G family were evaluated through a simulation study. Application of the sub-model to the Breaking strength data clearly showed its superiority over the other competing models.

Keywords: Entropy; new Weibull inverse Lomax; complete sample; Monte Carlo simulation; inverse lomax; inverse Lomax-G family.

Mathematical Subject Classification: 60E05, 62E15.

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1 Introduction

The distribution of Inverse Lomax (IL) is part of a distribution of Beta forms. Other family members include, among others, Pareto (1), logistics, Dagum, generalized second-type beta distributions, and Singh maddala [1]. In several fields, such as Actuarial Science and Economics, IL distribution has since gained more attention (see [1]), Geophysical data (see [2]), Survival analysis (see [3]), and Medical Science (see [4] and [5]).

Some attempts have been made to define new families of probability distributions which strengthen well-known distribution families while providing greater flexibility in practical data modeling. Following from the T-X approach by [6], we define the cumulative distribution function (cdf) of the new family of distributions as

$$F(x) = \int_{b}^{N(H(x))} l(t_1)dt_1 \tag{1.1}$$

where $N(H(x))$ is the function of the baseline cdf H(x) of any random variable (RV) X that satisfies the following conditions below:
(a) $N(H(x)) \in [b, c]$
(b) $N(H(x))$ Is non-decreasing, monotonically differentiable
(c) $N(H(x)) \Rightarrow$ b as x $\Rightarrow -\infty$ and $N(H(x)) \Rightarrow c$ as $x \Rightarrow \infty$. Let T_1 be a RV which is continuous with a probability density function (pdf) $l(t_1)$ defined on the closed interval [b, c].

Some of the generalized families of distributions based on this approach in the literature include: Weibull G family by [7], Lomax Generator of distributions by [8], Odd Generalized Exponential family by [9], Odd Lindley-G family by [10], Gompertz-G family by [11], Odd Frechet G family by [12], Power Lindley G family by [13], Topp Leone Exponentiated-G by [14], and Odd Chen-G family by [15].

Inverse Lomax (IL) distribution has both scale and shape parameters which makes it more flexible in modeling datasets. In order to increase its flexibility and usage, we wish to generalize the IL model. The probability density function (pdf) and cdf of the IL distribution are given by

$$h(x; \alpha, \beta) = \frac{\alpha\beta}{x^2}\left(1 + \frac{\beta}{x}\right)^{(-\alpha-1)} \tag{1.2}$$

$$H(x; \alpha, \beta) = \left(1 + \frac{\beta}{x}\right)^{-(\alpha)}; \qquad x > 0, \alpha, \beta > 0 \tag{1.3}$$

The objective of this paper is to propose a new family of distributions called the Inverse Lomax-G family of distributions which has the capacity of providing more robust compound probability distributions when modeling real life data set. This new family adds two additional parameters to the baseline distribution. This article is structured as follows: The Inverse Lomax-G family (IL-G) is defined in Sec. 2. In Sec. 3 a special sub-model of the IL-G is derived. A mixture representation of the cdf of the IL-G is presented in Sec. 4, while some of the IL-G's properties including the quantile function, order statistics, moments, moment generating function, and Entropies are discussed in Sec. 5. In Sec. 6, the estimation of the parameters of the IL-G is conducted using the Maximum Likelihood method. A Monte Carlo simulation study is used to estimate the bias and mean squared error of the MLE estimates of the parameters of the sub-model in Sec. 7. The application of the Inverse Lomax Weibull (ILW) and its competitors to the Breaking strength data is discussed in Sec. 8, while Sec. 9 conludes the paper.

2 The Inverse Lomax-G Family

In this section, we derive the distribution of the Inverse Lomax-G Family. The pdf, cdf, hazard function (hf), reversed hazard function, survival function, and cumulative hf are displayed.

Let $H(x;\upsilon)$ and $h(x;\upsilon)$ be the baseline cdf and pdf, and υ be a vector of parameters, let $l(t_1)$ be as defined in Eqt. 1.2, then the cdf $F(x;\alpha,\vartheta) = F(x;\vartheta)$ of the IL-G family of distribution is defined as

$$F(x;\vartheta) = \int_0^{\frac{H(x;\upsilon)}{1-H(x;\upsilon)}} l(t_1)dt_1 = \left(1 + \frac{\beta\bar{H}(x;\upsilon)}{Hx;\upsilon)}\right)^{-\alpha}; \qquad x > 0, \alpha, \beta, \upsilon > 0 \tag{2.1}$$

where $\vartheta = (\alpha,\beta,\upsilon)^T$, $\bar{H}(x;\upsilon) = 1 - H(x;\upsilon)$ and also α and β are the two additional parameters that are added to make the baseline distribution much more flexible. The corresponding pdf $f(x;\vartheta)$ of IL-G family obtained by differentiating Eqt. 2.1 as given below

$$f(x;\vartheta) = \frac{\beta\alpha h(x;\upsilon)}{[H(x;\upsilon)]^2}\left(1 + \frac{\beta\bar{H}(x;\upsilon)}{Hx;\upsilon)}\right)^{-(1+\alpha)} \tag{2.2}$$

The hazard function (hf), reversed hazard function rhf(x), survival function (s), and cumulative hazard functions C(x) are given below:

$$hf(x;\vartheta) = \frac{\alpha\beta h(x;\upsilon)}{[H(x;\upsilon)]^2}\left(1 + \frac{\beta\bar{H}(x;\upsilon)}{Hx;\upsilon)}\right)^{-(1+\alpha)}\left[1 - \left(1 + \frac{\beta\bar{H}(x;\upsilon)}{Hx;\upsilon)}\right)^{-\alpha}\right]^{-1} \tag{2.3}$$

$$r(x;\vartheta) = \frac{\alpha\beta h(x;\upsilon)}{[H(x;\upsilon)]^2\left(1 + \frac{\beta\bar{H}(x;\upsilon)}{H(x;\upsilon)}\right)} \tag{2.4}$$

$$s(x;\vartheta) = 1 - \left(1 + \frac{\beta\bar{H}(x;\upsilon)}{Hx;\upsilon)}\right)^{-\alpha} \tag{2.5}$$

$$C(x;\vartheta) = \int_{-\infty}^{x} p(q)dq = -log[s(x)] = -log\left[1 - \left(1 + \frac{\beta\bar{H}(x;\upsilon)}{Hx;\upsilon)}\right)^{-\alpha}\right] \tag{2.6}$$

The quantile function (qf) of IL-G family can be derived by inverting Eqt. 2.1 as given below

$$Q(U) = H^{-1}\left\{\frac{\beta}{U^{(-\frac{1}{\alpha})} + \beta - 1}\right\} \tag{2.7}$$

where $H^{-1}(.)$ is quantile function (qf) of any baseline distribution, U is uniformly distributed i.e $U \sim \mathcal{U}(0,1)$, and Eqt. 2.7 can be used to draw samples for the purpose of simulation studies.

3 Special Sub model

Suppose that the parent distribution is Weibull. Then $h(x;\theta,\lambda) = \lambda\theta x^{\theta-1}\exp\{-\lambda x^\theta\}$ and $H(x;\theta,\lambda) = 1 - \exp\{-\lambda x^\theta\}$. With $x > 0$, $\lambda, \theta > 0$, the Inverse Lomax Weibull (ILW) distributon has the cdf given as

$$F_{ILW}(x) = \left(1 + \frac{\beta}{\exp\{\lambda x^\theta\} - 1}\right)^{-\alpha} \tag{3.1}$$

The ILW distribution becomes Inverse Lomax Exponential (ILExp) distribution when $\theta = 1$ and an Inverse Lomax Rayleigh (ILR) distribution when $\theta = 2$. The corresponding pdf of Eqt. (3.1) is given below:

$$f_{ILW}(x) = \frac{\alpha\beta\lambda\theta x^{\theta-1}\exp\{-\lambda x^\theta\}}{[1 - \exp\{-\lambda x^\theta\}]^2}\left(1 + \frac{\beta}{\exp\{\lambda x^\theta\} - 1}\right)^{-(1+\alpha)} \tag{3.2}$$

The quantile function, rhf(x), C(x), and hf(x), are given by

$$Q(U) = \left[\frac{log\left[\frac{U^{-(\frac{1}{\alpha})}+\beta-1}{U^{-(\frac{1}{\alpha})}-1}\right]}{\lambda} \right]^{\frac{1}{\theta}} \tag{3.3}$$

$$h_{ILW}(x) = \frac{\theta\lambda\alpha\beta x^{\theta-1}\exp\{-\lambda x^{\theta}\}\left(1+\frac{\beta}{\exp\{\lambda x^{\theta}\}-1}\right)^{-(1+\alpha)}}{\left[1-\left(1+\frac{\beta}{\exp\{\lambda x^{\theta}\}-1}\right)^{-\alpha}\right][1-\exp\{-\lambda x^{\theta}\}]^2} \tag{3.4}$$

$$H_{ILW}(x) = -log\left[\left(1+\frac{\beta}{\exp\{\lambda x^{\theta}\}-1}\right)^{-\alpha}\right] \tag{3.5}$$

$$r_{ILW}(x) = \frac{\theta\beta\alpha\lambda x^{\theta-1}\exp\{-\lambda x^{\theta}\}\left(1+\frac{\beta}{\exp\{\lambda x^{\theta}\}-1}\right)^{-(1+\alpha)}}{\left[\left(1+\frac{\beta}{\exp\{\lambda x^{\theta}\}-1}\right)^{-\alpha}\right][1-\exp\{-\lambda x^{\theta}\}]^2} \tag{3.6}$$

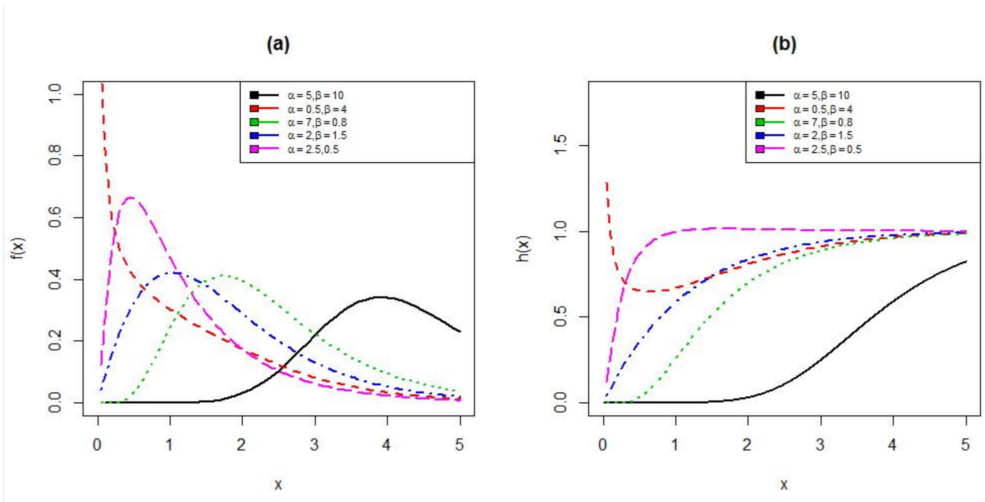

Fig. 1. Density and hf plots of ILW distribution with $\theta = 1$ and $\lambda = 1$ and varying α and β

Fig. 1 illustrates different density and hazard forms of the ILW distribution. By fixing the parameters of the baseline distribution and varying the additional two parameters, α and β, the plots show the additional flexibility induced by the addition of these two parameters to the weibull distribution.

4 Mixture Representations

Here, we present the power series expansion of the IL-G family by expanding Eqt. 2.1.
using the binomial expansion

$$(1+x)^{-t} = \sum_{c=0}^{\infty}\left(\begin{array}{c} -t \\ c \end{array}\right)x^{-t-c}$$

(mathworld.wolfram.com/BinomialCoefficient.html)

$$F(x;\vartheta)=\left(1+\frac{\beta\bar{H}(x;\upsilon)}{Hx;\upsilon)}\right)^{-\alpha}=\sum_{i=0}^{\infty}\left(\begin{array}{c}-\alpha\\ i\end{array}\right)\left[\frac{\beta\bar{H}(x;\upsilon)}{Hx;\upsilon)}\right]^{-(\alpha+i)} \tag{4.1}$$

$$F(x;\vartheta)=\sum_{i=0}^{\infty}\left(\begin{array}{c}-\alpha\\ i\end{array}\right)\beta^{-(\alpha+i)}\bar{H}(x;\upsilon)^{-(\alpha+i)}H(x;\upsilon)^{(\alpha+i)} \tag{4.2}$$

since

$$\bar{H}(x;\upsilon)^{-(\alpha+i)}=\sum_{j=0}^{\infty}\frac{\Gamma(\alpha+j+i)}{j!\Gamma(\alpha+i)}H(x;\upsilon)^{j}$$

Then, Eqt. (4.2) can also be written as

$$F(x;\vartheta)=\sum_{i,j=0}^{\infty}\left(\begin{array}{c}-\alpha\\ i\end{array}\right)\frac{\Gamma(\alpha+j+i)}{j!\Gamma(\alpha+i)}\beta^{-(\alpha+i)}H(x;\upsilon)^{(\alpha+i+j)} \tag{4.3}$$

$$F(x;\vartheta)=\sum_{i,j=0}^{\infty}\sigma_{(i,j)}\Pi_{(i,j)}(x;\upsilon) \tag{4.4}$$

where $\sigma_{(i,j)}=\left(\begin{array}{c}-\alpha\\ i\end{array}\right)\frac{\Gamma(\alpha+j+i)}{j!\Gamma(\alpha+i)}\beta^{-(\alpha+i)}$ and $\Pi_{(i,j)}(x;\upsilon)$ Is the exp-G family cdf with parameter power $(i+\alpha+j)$. The corresponding IL-G pdf is given by

$$f(x;\vartheta)=\sum_{i,j=0}^{\infty}\sigma_{(i,j)}\pi_{(i,j)}(x;\upsilon) \tag{4.5}$$

where $\pi_{(i,j)}(x;\upsilon)=(\alpha+i+j)h(x;\upsilon)H(x;\upsilon)^{(\alpha+i+j-1)}$.

5 Mathematical Properties of IL-G

We derived some of the mathematical properties of the IL-G family.

5.1 Moments and Moments Generating Function (mgf)

Suppose the random variable X comes from IL-G family with parameter space ϑ, then the r^{th} moment is given by

$$E(X^{r})=\int_{0}^{\infty}x^{r}f(x)dx=\int_{0}^{\infty}x^{r}\sum_{i,j=0}^{\infty}\sigma_{(i,j)}\pi_{(i,j)}(x;\upsilon)dx \tag{5.1}$$

$$=\sum_{i,j=0}^{\infty}\sigma_{(i,j)}\int_{0}^{\infty}x^{r}(\alpha+j+i)h(x;\upsilon)H(x;\upsilon)^{(i+j+\alpha-1)}dx \tag{5.2}$$

$$=\sum_{i,j=0}^{\infty}\sigma_{(i,j)}E(Z^{r}_{(i,j)}) \tag{5.3}$$

where $Z^{r}_{(i,j)}$ Denotes the power-parameter Exp-G distribution $(\alpha+i+j-1)$.

The mgf of IL-G RV is defined as

$$M_{X}(q)=\int_{-\infty}^{\infty}\exp\{qx\}f(x)dx \tag{5.4}$$

By expanding Eqt. 5.4 using taylor series,

$$M_X(q) = \sum_{r=0}^{\infty} \frac{q^r}{r!} \int_{-\infty}^{\infty} x^r f(x) dx \tag{5.5}$$

Substituting Eqt. (5.3) in to the defination of $M_X(q)$ yields

$$M_X(q) = \sum_{r=0}^{\infty} \frac{q^r}{r!} E(X^r) \tag{5.6}$$

5.2 Order Statistics

Order statistics are used in other areas of statistical theory and procedures to classify outliers in statistical quality control systems. We derive the pdf from the p^{th} order statistic of the IL-G family of distributions in closed form. Suppose $X_1, X_2, X_3, X_4 \dots X_n$ is a random sample from a distribution with pdf $f(x)$ and let $X_{1:n}, X_{2:n}, X_{3:n}, X_{4:n} \dots X_{n:n}$ denotes the corresponding order statistics obtained from this sample. Then

$$f_{p:n}(x;\vartheta) = \frac{f(x)}{B_1(p, n-p+1)} F(x)^{p-1} [1 - F(x)]^{n-p} \tag{5.7}$$

where $f(x)$ and $F(x)$ are the pdf and CDF of the IL-G distribution as in Eqt. (2.2) and Eqt. (2.1) respectively. Using the fact that

$$[1 - F(x)]^{n-p} = \sum_{i=0}^{n-p} (-1)^i \binom{n-p}{i} F(x)^i \tag{5.8}$$

By substituting Eqt. (5.8) in Eqt. (5.7) we have

$$f_{p:n}(x;\vartheta) = \frac{f(x)}{B_1(p, n-p+1)} \sum_{i=0}^{n-p} (-1)^i \binom{n-p}{i} F(x)^{i+p-1} \tag{5.9}$$

also,

$$F(x)^{i+p-1} = \sum_{j=0}^{\infty} \binom{-\alpha(i+p-1)}{j} \beta^{-\alpha(i+p-1)-j} \left[\frac{\bar{H}(x;\upsilon)}{H(x;\upsilon)}\right]^{-\alpha(i+p-1)-j} \tag{5.10}$$

Eqt. (5.9) can be simplified as

$$f_{p:n}(x;\vartheta) = \Omega_{(ijk)} h(x;\upsilon) H(x;\upsilon)^{\alpha(i+p)+j+k+l-1} \tag{5.11}$$

where

$$\Omega_{(ijk)} = \frac{\alpha\beta^{-[a\alpha(i+p)+k+k+j]}}{B_1(p, n-p+1)} \sum_{i=0}^{n-p} \sum_{j,k,l=0}^{\infty} (-1)^i \binom{n-p}{i} \binom{-\alpha(p+i-1)}{j} \binom{-(1+\alpha)}{k} \frac{\Gamma(\alpha(p+i)-k-j+l-1)}{\Gamma(\alpha(i+p)-k-j-1)} \tag{5.12}$$

$B_1(,)$ is a beta function, and
$h(.)$ also $H(.)$ are the baseline pdf and cdf respectively.

5.3 Entropy

Here, we consider the Rényi entropy by [16] and Shannon entropy by [17]. One unknown variance measure is called the entropy of a random variable X. The Rényi entropy for IL-G random variable is provided by

$$I_R(\eta) = \frac{1}{(1-\eta)} log \left[\int_0^{\infty} f^{\eta}(x) dx\right], \eta > 0 \, and \, \eta \neq 1 \tag{5.13}$$

where from Eqt. (2.2)
$f^{\eta}(x) = \left[\frac{h(x;\upsilon)\alpha\beta}{[H(x;\upsilon)]^2}\left(1+\frac{\beta\bar{H}(x;\upsilon)}{Hx;\upsilon)}\right)^{-(1+\alpha)}\right]^{\eta}$
Then, by expanding $f^{\eta}(x)$ using a similar process as in Sec. (4) and some simplifications, yields

$$I_R(\eta) = \frac{1}{(1-\eta)}log\left[\sum_{i,j=0}^{\infty}\rho_{(i,j)}\int_0^{\infty}h(x;\upsilon)^{\eta}H(x;\upsilon)^{\eta(\alpha-1)+i+j}dx\right] \tag{5.14}$$

where $\rho_{(i,j)} = \alpha^{\eta}\beta^{-(\eta\alpha+i)}\begin{pmatrix}-\eta(1+\alpha)\\ i\end{pmatrix}\frac{\Gamma(\eta(1+\alpha)+j-i)}{j!\Gamma(\eta(1+\alpha)-i)}$.
Shannon Entropy when $\eta\uparrow 1$, is a special case of Rényi entropy given by

$$E\{-log\,[f(x;\vartheta)]\} = -log(\alpha\beta) + E\left[-log\left[\frac{h(x;\upsilon)}{H(x;\upsilon)^2}\right]\right] + (1+\alpha)E\left[log\left(1+\frac{\beta\bar{H}(x;\upsilon)}{Hx;\upsilon)}\right)\right] \tag{5.15}$$

6 Estimation

We present the maximum likelihood estimates of the parameters of the IL-G distribution in this section. Let $x_1, x_2, x_3, \ldots, x_n$ be the observed values of n observations independently drawn from the ILG family with ϑ. The log-likelihood (ll) function for ϑ denoted by $l(\vartheta)$ can be expressed as

$$l(\vartheta) = nlog(\alpha\beta) + \sum_{i=1}^{n}log\,(h(x_i;\upsilon)) - 2\sum_{i=1}^{n}log\,(H(x_i\upsilon)) - (1+\alpha)\sum_{i=1}^{n}log\left(1+\frac{\beta\bar{H}(x;\upsilon)}{Hx;\upsilon)}\right) \tag{6.1}$$

haven taken the partial derivatives of Eqt. (6.1) with respect to α, β, and υ, we derived $U(\vartheta)$ i.e the Score Vector components are as follows

$$U_{\alpha}(\vartheta) = \frac{n}{\alpha} - \sum_{i=1}^{n}log\left(1+\frac{\beta\bar{H}(x;\upsilon)}{Hx;\upsilon)}\right) \tag{6.2}$$

$$U_{\beta}(\vartheta) = \frac{n}{\beta} + \sum_{i=1}^{n}\frac{-(1+\alpha)\frac{\bar{H}(x_i;\upsilon)}{H(x_i;\upsilon)}}{\left(1+\frac{\beta\bar{H}(x;\upsilon)}{Hx;\upsilon)}\right)} \tag{6.3}$$

$$U_{\upsilon}(\vartheta) = \sum_{i=1}^{n}\left(\frac{h^1(x_i;\upsilon)}{h(x_i;\upsilon)}\right) - 2\sum_{i=1}^{n}\left(\frac{h(x_i;\upsilon)}{H(x_i;\upsilon)}\right) + \sum_{i=1}^{n}\left\{\frac{(1+\alpha)\beta h(x;\upsilon)}{H^2(x_i;\upsilon)\left(1+\frac{\beta\bar{H}(x;\upsilon)}{Hx;\upsilon)}\right)}\right\} \tag{6.4}$$

Setting Eqts. (6.2, 6.3, and 6.4) to zero and also solving simultaneously yields the MLE $(\hat{\vartheta}) = (\hat{\alpha}, \hat{\beta}, \hat{\upsilon})$ of ϑ. However, these equations cannot be easily solved analytically. Therefore, statistical software is employed to solve the equations numerically through iterative methods.

7 Simulation Studies

A Monte Carlo simulation is conducted and the results of the bias and mean squared error of the various estimated parameter values are presented in Tables 1 and 2. The Monte carlo simulation is described as follows:

(a) For known parameter values i.e $\vartheta = (\alpha, \beta, \lambda, \theta)^T$, samples of different sizes from the Inverse Lomax Weibull distribution were generated with small parameter values ($\alpha = .5$, $\beta = .8$, $\lambda = .7$, and $\theta = .6$) and with relatively big parameter values ($\alpha = 5$, $\beta = 10$, $\lambda = 7$, and $\theta = 6$) using the quantile function defined in Eqt. (3.3).

(b) Using the maximum likelihood method, we compute the MLE of $\hat{\alpha_i}$, $\hat{\beta_i}$, $\hat{\lambda_i}$, and $\hat{\theta_i}$ for the i^{th} replicate.

(c) Steps (a) and (b) are replicated N=500 times.

(d) The bias and MSE for each sample size n are computed as

$$\hat{\vartheta} = \frac{1}{N}\sum_{i=1}^{N}\hat{\vartheta}_i, \qquad Bias(\hat{\vartheta}) = (\hat{\vartheta} - \vartheta), \qquad MSE(\hat{\vartheta}) = \frac{1}{N}\sum_{i=1}^{N}(\hat{\vartheta}_i - \vartheta)^2 \tag{7.1}$$

where $\hat{\vartheta}_i = (\hat{\alpha_i}, \hat{\beta_i}, \hat{\lambda_i}, \hat{\theta_i})$ are the mle for the i^{th} replicate. For both small and big parameter values, the sample size $n = 30, 50, 75, 200, 300, 500$ are used to evaluate the behaviour of the bias, variance, and mean squared error as the sample size increases.

It is clear from the simulation study that both bias and mean squared error decrease as the sample size increases for both small and relatively big parameter values. Also, for relatively large sample size, the bias appears very negligible for both small and relatively big parameter values, as shown in Table 1. and Table 2.

Table 1. Simulation results for the ILW distribution with small parameter values

n	Properties	α=.5	β=.7	λ=.9	θ=.4
30	Bias	0.3533	1.5359	0.5527	0.1136
	MSE	1.1681	9.7608	1.3789	0.2539
	Var.	1.0433	7.4016	1.0734	0.2411
	Est.	0.8533	2.3359	1.2527	0.7136
50	Bias	0.2538	1.2946	0.5639	0.0135
	MSE	0.5421	7.3381	1.3247	0.1123
	Var.	0.4777	5.6622	1.0067	0.1121
	Est.	0.7538	2.0946	1.2639	0.6135
75	Bias	0.1577	1.0535	0.4876	-0.0073
	MSE	0.1936	5.1485	1.0184	0.0836
	Var.	0.1687	4.0386	0.7806	0.8433
	Est.	0.6577	1.8535	1.1876	0.5927
200	Bias	0.1252	0.8796	0.4979	-0.0641
	MSE	0.0717	3.0456	0.8092	0.0372
	Var.	0.056	2.2719	0.5612	0.0331
	Est.	0.6252	1.6796	1.1979	0.5359
300	Bias	0.0942	0.6233	0.3792	-0.0475
	MSE	0.0464	1.8196	0.5768	0.032
	Var.	0.0375	1.4311	0.4329	0.0298
	Est.	0.5942	1.4233	1.0792	0.5525
500	Bias	0.0816	0.4833	0.3331	-0.0504
	MSE	0.0464	0.9801	0.4249	0.0247
	Var.	0.0375	0.7465	0.3139	0.0222
	Est.	0.5942	1.2833	1.0331	0.5496

Table 2. Simulation results for the ILW distribution with big parameter values

n	Properties	α=5	β=10	λ=7	θ=6
30	Bias	2.6185	-4.9519	-0.6999	3.9134
	MSE	30.8643	49.8567	3.7481	37.5244
	Var.	24.0076	25.3349	3.2581	22.2094
	Est.	7.6185	5.048	6.3	9.9134
50	Bias	3.0295	-4.9676	-0.8143	3.2122
	MSE	32.48	47.8544	3.2679	29.6492
	Var.	23.3021	23.1776	2.6048	19.3307
	Est.	8.0295	5.0324	6.1857	9.2122
75	Bias	3.2687	-4.8458	-0.7847	2.4964
	MSE	32.3415	46.8771	2.7158	19.5839
	Var.	21.6569	23.3953	2.0999	13.3521
	Est.	8.2687	5.1542	6.2153	8.4964
200	Bias	2.9869	-4.6932	-0.7209	1.6219
	MSE	25.957	43.4982	1.7507	9.7688
	Var.	17.0351	21.4725	1.2309	7.1383
	Est.	7.9869	5.3068	6.2791	7.6219
300	Bias	2.6608	-4.3656	-0.6686	1.3504
	MSE	20.1019	42.8896	1.4626	6.6039
	Var.	13.0219	23.8313	1.0156	4.7804
	Est.	7.6608	5.6344	6.3314	7.3504
500	Bias	1.5085	-2.4509	-.3457	.6299
	MSE	8.8700	32.4786	.6288	2.0385
	Var.	6.5943	26.4716	.5092	1.6417
	Est.	6.5085	7.5491	6.6543	6.6299

8 Application

We demonstrate the usefullness of the ILW distribution to the breaking strength of 100 Yarn as reported by [10]. The data-set consists of 63 measurements of the strengths of 1.5 cm glass fibres, which were initially collected by United Kingdom National Physical Laboratory staff is as presented below:

0.55, 0.74, 0.77, 0.81, 0.84, 1.24, 0.93, 1.04, 1.11, 1.13, 1.30, 1.25, 1.27, 1.28,1.29, 1.48, 1.36, 1.39, 1.42, 1.48, 1.51, 1.49, 1.49, 1.50, 1.50,1.55, 1.52, 1.53, 1.54, 1.55, 1.61, 1.58,1.59, 1.60, 1.61, 1.63,1.61, 1.61, 1.62, 1.62, 1.67, 1.64, 1.66, 1.66, 1.66, 1.70, 1.68,1.68, 1.69, 1.70, 1.78, 1.73, 1.76, 1.76, 1.77, 1.89, 1.81, 1.82,1.84, 1.84, 2.00, 2.01, 2.24.

We used a maxLik package developed by [18] in R and used by [19]. The goodness-of-fit (gof) statistics used to compare the models' performance are the Akaike Information Criterion (AIC), AIC with correction for small sample sizes (AICc), and the Bayesian Information Criterion (BIC). Smaller values of the statistics i.e AIC, AICc, and BIC statistics indicates better model fittings. The competing models (distribution) that are used with the ILW are:

(a) The Weibull-Weibull (ww) by [7] with cdf

$$F_{ww}(x;\alpha,\beta,\theta) = 1 - \exp\left\{-\alpha\left(\exp\left\{\theta x^{\lambda}\right\} - 1\right)^{\beta}\right\} \qquad x > 0$$

(b) The Odd Lindley Weibull (OddLW) by [10] with cdf

$$F_{OddLW}(x;\alpha,\beta,\lambda) = (1+\alpha)^{-1}\exp\left\{-\alpha\left(\alpha + \exp\left\{(\lambda x)^{\beta}\right\}\right)\right\}$$
$$\left\{(1+\alpha+\alpha^2)\exp\left\{\alpha\exp\left\{(\lambda x)^{\beta}\right\}\right\} - \exp\left\{\alpha(1+\alpha)\right\}\left[1+\alpha\exp\left\{(\lambda x)^{\beta}\right\}\right]\right\} \qquad x > 0$$

(c) The Lomax Weibull (LW) by [8] with cdf

$$F_{LW}(x;\alpha,\beta,\lambda,\theta) = 1 - \left\{\frac{\beta}{\beta + (\lambda x)^{\theta}}\right\}^{\alpha} \qquad x > 0$$

(d) The baseline Weibull (w) with cdf

$$F_w(x;\lambda,\theta) = 1 - \exp\left\{-\lambda x^{\theta}\right\} \qquad x > 0$$

The comperators are all extensions of the baseline weibull distribution function, with Weibull-Webull and Lomax Weibull each having four parameters like the new Inverse Lomax Weibull distribution.

The histogram of the Breaking Strength data, and estimated pdf of the ILW and the other competing distributions are presented in Fig. 2. The graph confirms the results of the AIC, AICc, and BIC statistics presented in Table 3 that the ILW distribution fitted the Breaking Strength data better than the other distributions.

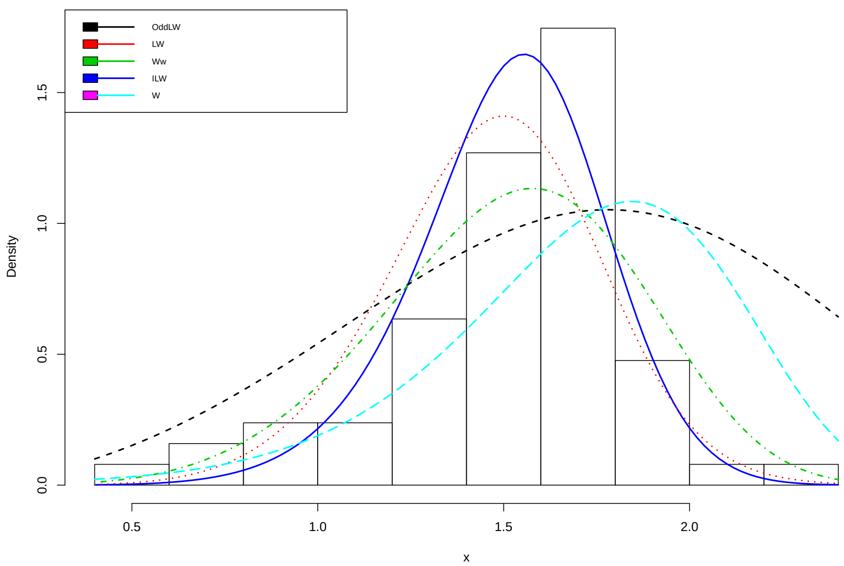

Fig. 2. Fitted distributions to Breaking Strength data

As shown in Table (3), the ILW distribution apears to be the best with minimum AIC values, and AICc. Its ranked number 1 outperforming the OddLW by [10] that also used the same data set.

Table 3. MLEs and Standard Errors in parenthesis and -ll, AIC, BIC, AICc for the data set

Model	α	β	λ	θ	-ll	AIC	BIC	AICc
ILW	0.8702	12.3997	0.4801	3.6445	11.8329	31.6658	40.2383	32.3555
	(0.2231)	(7.1987)	(0.2082)	(0.6083)	–	–	–	–
OddLW	0.0491	–	1.1021	0.4921	14.1935	34.3874	40.8164	34.7938
	(0.0871)	–	(0.5271)	(0.4941)	–	–	–	
W	–	–	5.7811	1.6281	15.2071	34.1412	40.9001	34.6142
	–	–	(0.5761)	(0.0371)	–	–	–	–
WW	3.2796	2.4913	0.1895	1.8858	14.6773	37.3546	45.9271	38.0443
	(6.1346)	(12.3435)	(1.1737)	(9.6246)	–	–	–	–
LW	11.9908	3.5595	6.2205	0.51042	15.4438	38.8877	47.4601	39.5774
	(0.0001)	(1.2542)	(0.6532)	(0.0287)	–	–	–	–

9 Conclusion

In this paper, we propose and evaluate a new class of distributions called the Inverse Lomax-G (IL-G) Family of Distributions. This family can extend several widely known models. For illustrative purpose, we considered Weibull as base line distribution. Using power series expansion, we derived some of its properties such as an expansion for density function. Some of the derived properties include Moments, Moment generating function, quantile function, order statistics, and entropies. The ILW (by taking Weibull as baseline distribution) parameters were estimated using MLE. The Monte Carlo Simulation results showed that the bias of the MLEs became negligible when sample sizes are greater than 100. The analysis of the Breaking Strength data indicated the superiority of this family of distribution over the competing families.

Competing Interests

Authors have declared that no competing interests exist.

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