



Aspects of Modern Systemic Approach (III): Implications of Random Processes in the Study of Dynamic Systems

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Abstract

The purpose of this study is to familiarize the reader with the diversity of concepts (notions) and stages of development specific to Probability Theory, with the definition and characterization of variables, vectors and random processes, respectively with the most important elements that give random processes. Among these we mention the distribution function, the probability density function, the statistical moments of a random process, the temporal averages, and respectively the correlation (and intercorrelation) of a random signal (process). Also, the implications of random processes in the study of dynamical systems are reviewed, as well as a series of applications specific to the analysis of dynamic behavior.

Keywords: Probability; random processes; mathematical representations; dynamic system.

1 Introduction

It can be considered that the beginning of probability theory is in the seventeenth century, in a series of letters from 1654 between Blaise Pascal (1623-1662) [1,2] and Pierre de Fermat (1601-1665) [3], in which

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they solved the problem of the unfinished game, a problem related to gambling and raised Pascal by Antoine Gombaud-Cavalier de Méré (1607-1684) [4]. The problem of the unfinished game requires the determination of the pot split when a multi-round game must be abandoned before it is over.

Before the correspondence between Pascal and Fermat, Girolamo Cardano (1501-1576) [5], in his book *Book of Gambling (Liber de ludo aleae)*, published the first scientific study on the roll dice, based on the hypothesis that some fundamental principles are governing the probability of obtaining certain results. The mathematics created by Pascal and Fermat, which seemed dysfunctional in the chaos and unpredictability of everyday life, received a substantial boost with the publication by John Graunt (1620-1674) [6] of the book *Natural and Political Observations Based on Mortuary Announcements*. In his paper, Graunt analyzes and organizes London death records from the 1660s to create a system to signal the onset and spread of the plague in London. This brochure can be considered as the cornerstone of modern statistics [7,8]. The techniques developed and used by Graunt in the paper made it possible to apply the theory developed by Pascal and Fermat, initially for gambling, in everyday life. For the first time, mankind had a means of making predictions and controlling it through decisions based on risk analysis.

In 1657, Christiaan Huygens (1629-1695) [9] wrote *On the Calculations of Gambling (De ratiociniis in ludo aleae)*, the first true work of probability theory in the modern sense. In this paper, Huygens establishes the fundamental rules of probability calculus, starting from the axioms of probability theory. Huygens also introduces the notion of hope (or expected gain, which is calculated by multiplying the probability of each outcome by the amount to be won, or lost - a negative gain - and summing all the results) and uses Graunt's tables to calculate the hope of life.

Jakob Bernoulli (1654-1705) [10] made a significant mathematical contribution by stating the "law of large numbers", an important consequence of probability theory. This law gives the mathematical formulation of the fact that the relative frequency of an event will predict the probability of its occurrence the more accurately the higher the number of observations from which the relative frequency is calculated. Bernoulli also showed that by choosing a sufficiently large sample, we can increase to any desired level the confidence that the probability calculated for the sample is equal to the true probability.

Abraham de Moivre (1667-1754) [11] showed that a set of random observations is distributed around an average value (today it is called normal distribution). Graphically, the resulting curve (results of observations vs. frequency of observations) has the shape of a bell. This shows that most of the results of the observations are grouped around the mean. From the average, the curve descends symmetrically.

At the beginning of the 19th century, Karl F. Gauss (1777-1855) [12] noted that the graphical representation of a series of results obtained by repeated measurement of the same size produces a curve that strikingly resembled de Moivre's bell. Gauss considered the bell curve to be a consequence of unavoidable measurement errors and used the normal distribution as a basis for estimating the probabilities of events (the results of observations).

On the road from solving the problem of the unfinished game to managing risk, the final mathematical step was taken by Thomas Bayes (1702-1761) [13] by finding an ingenious and extremely powerful mathematical formula that offers a method to correct a probability in the light of new information. Thus, if there is an estimate of the previous probability of occurrence of an event and if new (additional) information appears, a more accurate posterior probability of occurrence of that event can be obtained. Repeatedly applying Bayes' formula, each time new information is obtained, allows the correction of an unreliable previous probability so that, in the end, a very reliable posterior probability is obtained.

Numerous practical problems, associated with the phenomena of nature and society, from various fields of activity (electrical engineering, radio, data transmission, computers, information theory, reliability of systems, and others) lead to the study of random processes and phenomena. The evaluation of their chances of production is the object of the discipline of probability theory. Probability theory thus provides a mathematical model for describing and interpreting phenomena that have statistical regularity. Probability

theory allows modeling and quantifying the uncertainties that appear in the calculation models used to dynamical systems.

Through this paper, we want to make a brief foray, in the dynamic systems analysis from a random variables perspective. The dynamical behaviour of different systems, as well as their analysis and interpretation, were made mainly taking into account the relevant mentions of probability theory and various publications in the online environment.

2 Materials and Methods

The documentation for this paper started from the authors' concerns for dynamic systems, for the management, analysis, and interpretation of such systems in the context of Probability Theory, the focus being on implications of random processes in the study of dynamical systems. Having at hand a series of extremely relevant studies at the international level, and less national, in terms of the management of random processes, we decided to extrapolate these approaches to the relatively limited space of Systems Theory, where dynamic systems are in the foreground.

This is how we start our incursion in the random processes implications in dynamical systems identification, analysis, and behavior interpretation. There are a few studies that make direct reference to the management of systems, in general, or dynamic systems, in particular. The collection, analysis, and interpretation of data, mostly related to different processes - such as Markov, diffusion, Wiener, and white noise, was realized related to the Probability Theory.

3 Results and Discussion

3.1 Definition and characterization of random elements

The study of a system is of interest for understanding the relationships between its components or for predicting how it will work in new conditions. Sometimes it is possible to experiment with the system itself, but not always. Indeed, the system may not yet exist, but may only be in the hypothetical form or the design phase. Consequently, the study of systems is performed with the system model, according to the specific principles of the Systems Theory (ST) considered [14-16]. ST involves the interdisciplinary study of the abstract organization of phenomena, regardless of their substance, their type, the spatial or temporal scale of their existence. Systems theory also investigates principles common to all complex entities, as well as models (generally mathematical) that can be used to describe these entities (systems).

The notion of system is common in various fields of technology and science, in economics, in nature and society. Such are the concepts of the social system, economic system, system of equations, physical system, dynamic system, information system, management system, automatic system, etc [17,18]. A system comprises multiple aspects (for example planning, specifications, analysis, design, implementation, deployment, structure, behavior, input data, and output data). The model of a system is needed to describe and represent these multiple aspects. Systems modeling is a basic principle in engineering and the social sciences.

A model is defined as a conceptual (abstract) representation of a system that artificially reproduces and describes the existing original system, which allows the study of the system, thus serving to know the properties of the original system and predict its behavior. A model is a schematic description of a system, theory, or phenomenon that explains its known or assumed properties and can be used for further study of its properties.

For many studies, it is necessary to consider only those aspects (or variables, in this case, variables specific to random processes) of the system that are relevant to the researched problem. These (variable) aspects are represented in the system model, and the model, by definition, is a simplified representation of the system.

On the other hand, the model must be sufficiently detailed to allow valid conclusions to be drawn when conducting experiments on the model, to know the properties of the real system. No model of a system will include all the features of the real system of interest and no model of a system should include all the entities that belong to the real system of interest.

Systems are often viewed or modeled as component blocks that have connections to each other. There are alternatives to representing a system as a single unit on a single level, or as a collection of subsystems that must be coordinated at the general "system level". This is an important modeling decision when the size of the system is large. In the vast field of science few features cannot be described in mathematical terms or few areas that cannot benefit as a result of understanding the models of some systems. The use of models and the modeling process has become an important tool in systems analysis in that it offers the possibility of exploring hypotheses that cannot be easily tested by field or laboratory experiments.

Representations of the term model have been proposed by various authors. Jackson et al. (2000) propose an interpretation of the model by the idea of a particular representation of an idea or a condition, which varies in complexity from the simple form of assigning an action on a subject to the description of processes through mathematical equations. From the perspective of the author's Yu et al. the model is the formal description of the essential elements of a problem within a system of interest; Seppelt (2003) notes that models are tools that help to understand how processes work and regularly allow testing hypotheses, and Gillman (2009) offers a simplified version, defining the model as a representation of reality.

The choice to use a deterministic or stochastic model is expressed depending on the purpose and objectives of the modeling project. From the complexity considered in the model development process, in the case of deterministic ones it is reduced because it requires only the estimation of constants, while in the case of stochastic ones it is necessary to specify the complete distribution of values assigned to random variables. Regarding their use, in this case, too a low degree of difficulty is attributed to deterministic models, because to predict a given situation it is necessary to run a single model, whereas in the case of stochastic models it is necessary to perform an average of the replica responses of the predictions.

A random variable is a quantity whose value cannot be predicted with sufficient accuracy before an observation is made. An alternative definition can be formulated as follows: a random variable is a function whose values are real numbers and depend on chance. If a numeric value is assigned to any event that may occur as a result of observation, the resulting set of possible numbers is a random variable. Next, some clarifications will be made regarding the implications of the theory of random processes in the study of dynamic systems, with emphasis on the methods of analyzing the behavior of mechanical dynamic systems, which involve random variables in the form of associated vibrations.

As a result, we can say that the methods of analyzing the behavior of mechanical dynamic systems, as well as the random vibrations associated with them, have experienced a continuous development in recent decades due to high needs to design structures and equipment with superior functional performance and high reliability. , particularly complex, to which they are subjected during operation. A common feature of these types of demands is the impossibility to describe their evolution, over time, in a deterministic way, due to the behavioral dynamics of the whole system (the black box concept).

The dynamic behavior of mechanical systems with random parameters is described by stochastic differential equations whose treatment depends essentially on how the random factors intervene:

- Differential equations with random initial conditions - an important role in statistical mechanics, statistical thermodynamics, a priori analysis of spacecraft trajectories;
- Differential equations with random coefficients - used in the study of systems whose parameters have imprecise values due to inherent material or workmanship imperfections;
- Differential equations in which the random part enters as a non-homogeneous term - representing the external perturbation applied to the system in the form of a random time function.

The last category of differential equations has the widest field of applications, being used in modeling the dynamic behavior of most mechanical structures encountered in practice (road and rail vehicles, ships, aircraft, civil and industrial construction, machinery, machine tools, etc.). Depending on the practical problem analyzed different combinations of how the random factors mentioned above can be used.

Another particularly important notion, to which I refer to the study of dynamic systems, is that of random processes. Random variables represent, under the conditions of a fixed event (according to the specific requirements of a reference probability field), a particular realization of the random process. Other notions that appear as the novelty and that also have a significant role in the study of linear dynamical systems, from the perspective of random process theory are the following, although they were treated only at a lapidary level:

- Random process distribution function - defined as an extension of the distribution function of a random variable, which has developed over time; as a result, its value at a point will be the probability that the value of a particular realization of the random process at a given time is less than or equal to the value of the point;
- The probability density function - which is derived from the distribution function;
- Statistical moments of a random process - defined by the temporal extension of the statistical moments (averages) of a random variable;
- The temporal averages of a random signal (process) - can only be defined for a particular realization of it, so the usual cases of interest are the temporal averages of order 1 (continuous component) and order 2 (average power);
- Correlation (and intercorrelation) of random processes - defined analogously to the case of random variables; and here the essential difference is determined by the introduction of the temporal dimension, besides, there will be two types of correlation functions, depending on the mediation used: statistical or temporal.

Within the random processes, a special place is occupied by the Markov processes characterized in that the appearance of a certain state is conditioned only by a certain number of previous states. If the number of these previous states is r , then it is a Markov process of order r . Markov processes of order 1 occupy an important place in the study of traffic processes that characterize the telecommunication network as a whole for example. It means that if a system is at times in limited states of time and if it behaves like a 1st order Markov system, then the probability that at a later time its state will be determined only by its state from the previous moment.

3.2 Implications of random processes in the study and behaviour analysis of dynamic systems

In many practical cases, the excitations acting on a structure are random, and the response of the structure can no longer be described, under these conditions, in a deterministic way. Thus, the excitation induced by the unevenness of the road on a car, the earthquakes, the turbulence in the air, or the water, the action of the wind, or the waves, are examples of excitations that by their nature have a random character.

The theory of random processes was first successfully applied to Einstein's modeling of the behavior of dynamical systems in 1905, which showed that the probability density of the random process describing Brownian motion satisfies the diffusion equation.

A stochastic process, or sometimes a random process, is the opposite of the deterministic process (or deterministic system) considered in probability theory. Instead of a single possible reality about how processes can evolve, in a stochastic process, there is uncertainty in the future evolution described by probability distributions. This means that although the initial condition (or starting point) is known, there are more possibilities to continue the process, but some ways are more likely than others. A stochastic process can be represented as a random function. In practical applications, the domain of definition of such a process is a time interval - called the time series - or a place of space - in this case, called a random field.

Let \mathbb{R}^n be the n -dimensional Euclidean space with norm $\|a\| = \left(\sum_{k=1}^n a_k^2\right)^{1/2}$, $a = [a_1, a_2, \dots, a_n]^T \in \mathbb{R}^n$ its elements, called vectors, being represented as a column-matrix. Consider a reference probability field (Ω, K, P) , where Ω is the set of elementary elements, K is α -algebra of parts of Ω (random events), and P a complete measure defined on K with the property $P(\Omega) = 1$ (probability).

We note by $V^n(\Omega)$ the set of random vectors defined on Ω with values in \mathbb{R}^n and by $V_T^n(\Omega)$ the set of random processes defined on $T \times \Omega$, with values in \mathbb{R}^n . A random vector process $x(t, \omega) \in V_T^n(\Omega)$ represents a family of random vectors from $V^n(\Omega)$, indexed by the set T . Independent variable $t \in T$ has the significance of time.

3.2.1 Markov processes

A random vector process $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T \in V_T^n(\Omega)$ is called the Markov process if for any finite set $\{t_1, t_2, \dots, t_k | t_i < t_{i+1}\} \subset T$, any Borelian set $A \subset \mathbb{R}^n$ and any $x_1, x_2, \dots, x_k \in \mathbb{R}$, we have

$$P\left(\left\{\omega \in \Omega | x(t_k, \omega) \in A\right\} \middle| \left\{\omega \in \Omega | x(t_1, \omega) = x_1, x_2, \dots, x(t_{k-1}, \omega) = x_{k-1}\right\}\right) = P\left(\left\{\omega \in \Omega | x(t_k, \omega) \in A\right\} \middle| \left\{\omega \in \Omega | x(t_{k-1}, \omega) = x_{k-1}\right\}\right) \quad (1)$$

where for two random events $x_1, x_2, \dots, x_k \in \mathbb{R}$ represents the conditional probability of E_1 in relation to E_2 , defined by

$$P(E_1 | E_2) = \frac{P(E_1 \cup E_2)}{P(E_2)} \quad (2)$$

Relation (1) shows that Markov processes enjoy the property that, once they reach a certain state, under certain conditions, their statistical distribution in the future no longer depends on how they reached that state. Moreover, since these processes are subject to the principle of causality (the future can be predicted based on the knowledge of the present), they play a particularly important role in the study of dynamic systems subjected to random excitations (disturbances).

Conditional probability in the right member of the relation (1) it is called the transition probability of the Markov process $x(t)$ and it is noted with $P(A, t | x', t')$, where $t' < t$. Assuming the probability of transition $P(A, t | x', t')$ has a density $p(x, t | x', t')$ so as to

$$P(A, t | x', t') = \int_A p(x, t | x', t') dx, \quad t' < t \quad (3)$$

The property (1) can be expressed in relation to the probability densities conditioned in the form

$$p(x_k, t_k | x_1, x_2, \dots, x_{k-1}, t_1, t_2, \dots, t_{k-1}) = p(x_k, t_k | x_{k-1}, t_{k-1}), \quad (4)$$

$$t_1 < t_2 < \dots < t_k$$

Conditional probability density $p(x, t | x', t')$, $t' < t$, is called the transition probability density of the Markov process $x(t)$ and together with the first order probability density $p(x, t)$ completely determines, from a statistical point of view, the considered random process. Applying the definition of conditional probability density and property (4), you can write the relation

$$\begin{aligned} p(x_1, x_2, \dots, x_k, t_1, t_2, \dots, t_k) &= p(x_k, t_k | x_1, \dots, x_{k-1}, t_1, \dots, t_{k-1}) \times p(x_1, \dots, x_{k-1}, t_1, \dots, t_{k-1}) = \\ &= p(x_k, t_k | x_{k-1}, t_{k-1}) \times p(x_{k-1}, t_{k-1} | x_1, \dots, x_{k-2}, t_1, \dots, t_{k-2}) \times p(x_1, \dots, x_{k-2}, t_1, \dots, t_{k-2}) = \\ &= \dots = p(x_k, t_k | x_{k-1}, t_{k-1}) \times p(x_{k-1}, t_{k-1} | x_{k-2}, t_{k-2}) \times \dots \times p(x_2, t_2 | x_1, t_1) \times p(x_1, t_1), \end{aligned} \quad (5)$$

$$t_1 < t_2 < \dots < t_k$$

whence it follows that for any $k \in \mathbb{N}$, order probability density k it can be expressed in terms of first-order probability density and transition probability density. For the Markov process $x(t)$ the relation can be written

$$p(x'', x, t'' | x', t') = \dots = p(x, t | x'', t'') p(x'', t'' | x', t'), \quad t' < t'' < t \quad (6)$$

hence by integration on \mathbb{R}^n , the Chapman-Kolmogorov equation is obtained

$$p(x, t | x', t') = \int_{\mathbb{R}^n} p(x, t | x'', t'') p(x'', t'' | x', t') dx'', \quad t' < t'' < t \quad (7)$$

which is satisfied by the transition probability density of a Markov process.

Depending on the values of the random vector $x(t') \in V^n$ (for any t' given), transition probability density $p(x, t | x', t')$ is a random variable belonging to V^1 , whose average is precisely the first-order probability density of $x(t)$, respectively

$$M[p(x, t | x', t')] = p(x, t) \quad (8)$$

Applying the average operator results

$$M[p(x, t | x', t')] = \int_{\mathbb{R}^n} p(x, t | x'', t'') M[p(x'', t'' | x', t')] dx'' \quad (9)$$

where the relation

$$p(x, t) = \int_{\mathbb{R}^n} p(x, t | x'', t'') p(x'', t'') dx'', t'' < t \quad (10)$$

is obtained from, that is, the first-order probability density of a Markov process satisfies the Chapman-Kolmogorov equation. But if the Markov process $x(t) \in V_{\mathbb{R}_+}^n$ it is stationary in the broadest sense, then

$$p(x, t) = p(x), \quad p(x_1, x_2, t_1, t_2) = p(x_1, x_2, t_2 - t_1) \quad (11)$$

and by virtue of the relation

$$p(x_2, t_2 | x_1, t_1) = \frac{p(x_1, x_2, t_1, t_2)}{p(x_1, t_1)} \quad (12)$$

it turns out that the transition probability density $p(x_2, t_2 | x_1, t_1)$ it just depends on the difference $t_2 - t_1$, where $t_1 < t_2$. As a result, it can be written that

$$p(x_2, t_2 | x_1, t_1) = p(x_2, t | x_1, 0) \quad (13)$$

where $t_1 < t_2$ and $t = t_2 - t_1$. In this case, the transition probability density is denoted by $p(x, t | x')$, and the Chapman-Kolmogorov equation presented in (7) becomes

$$p(x, t + \tau | x') = \int_{\mathbb{R}^n} p(x, t | x'') p(x'', \tau | x') dx'' \quad (14)$$

as it immediately follows from (7) placing τ and $t + \tau$ in the place of t'' and t .

In the case of a stationary Markov process in the broad sense, applying the relation (5), we obtain that this random process is also stationary in the narrow sense. Therefore, Markov processes are part of the category of random processes for which stationary in the broad sense implies stationary in the narrow sense. We mention that this category also includes random processes with normal distribution which also play a significant role in modeling dynamic systems subjected to random excitations (perturbations).

If $\lim_{x \rightarrow \infty} p(x, t | x') = p_s(x)$, then $p_s(x)$ is called the asymptotically stable stationary probability density of the Markov process $x(t)$, in relation to t . Also in relation to the conditional probability density $p(x, t | x')$ the conditioned moments of different orders of the Markov process $x(t)$ can be defined, respectively

$$m_{k_1, k_2, \dots, k_n}(t | x') = \int_{\mathbb{R}^n} x_1^{k_1} \cdot x_2^{k_2} \cdot \dots \cdot x_n^{k_n} p(x, t | x') dx, \quad (15)$$

$$x = [x_1, x_2, \dots, x_n]^T, \quad k_1, k_2, \dots, k_n \in \mathbb{N}$$

The vector of first-order conditioned moments (the vector of conditioned averages) of the Markov process $x(t)$ it is, in a particular case, given by the relation

$$m(t|x') = \int_{\mathbb{R}^n} xp(x, t|x')dx \quad (16)$$

and the conditional covariance matrix has the form

$$C(t|x') = \int_{\mathbb{R}^n} [x - m(t|x')][x - m(t|x')]^T p(x, t|x')dx \quad (17)$$

In case of asymptotically stable stationary probability density, the mean vector and the covariance matrix of the Markov process $x(t)$ have asymptotic expressions

$$\begin{aligned} m &= \int_{\mathbb{R}^n} xp_s(x)dx, \\ C(\tau) &= \iint_{\mathbb{R}^{2n}} [x - m][x' - m]^T p(x, \tau|x')p_s(x')dx dx' \end{aligned} \quad (18)$$

By virtue of the relationships presented above it follows that $\lim_{t \rightarrow \infty} m(t|x') = m$ and $\lim_{t \rightarrow \infty} C(t|x') = C(0)$, because $\lim_{\tau \rightarrow 0} p(x, \tau|x') = \delta(x - x')$. Matrix $C(0)$ it is called the instantaneous covariance matrix of the stationary vector random process $x(t)$ and it is particularly important in characterizing the response of dynamic systems with random excitations, because it contains useful information on the intensity of the response and the statistical dependence between the simultaneous values of its components.

3.2.2 Diffusion processes

A Markov process $x(t) \in V_{\mathbb{R}_+}^n$ it is called the diffusion process, if the following conditions are met (fully met):

i. For any $x' \in \mathbb{R}^n$ and $\varepsilon > 0$

$$\int_{\|x-x'\|<\varepsilon} p(x, t|x', t')dx = 0(t-t') \quad (19)$$

uniform in relation with $t' < t$ and $x, x' \in \mathbb{R}^n$.

ii. There is the vector $a(x, t) = [a_1(x, t), a_2(x, t), \dots, a_n(x, t)]^T$ and matrix

$$B(x, t) = [b_{ij}(x, t)], \quad i, j = 1, 2, \dots, n, \text{ so for any } x \in \mathbb{R}^n \text{ and } \varepsilon > 0$$

$$\int_{\|x-x'\|<\varepsilon} (x-x')p(x, t|x', t')dx = a(x', t)(t-t') + 0(t-t') \quad (20)$$

$$\int_{\|x-x'\|<\varepsilon} (x-x')(x-x')^T p(x,t|x',t') dx = B(x',t)(t-t') + 0(t-t') \quad (21)$$

uniform in relation with $t' < t$ and $x, x' \in \mathbb{R}^n$.

The vector $a(x, t)$ is called the transfer vector, and the matrix $B(x, t)$ it is called the diffusion matrix of the vector random process $x(t)$. The name diffusion processes is based on the fact that random processes with the aforementioned properties have been used in modeling physical diffusion processes in inhomogeneous environments. In such a modeling, the random process $x(t) \in V_{\mathbb{R}^3}$ represents the position vector of a microscopic particle suspended in a liquid, the vector $a(x, t)$ defines the velocity of the liquid at the point x at the moment t , and the matrix $B(x, t)$ defines the properties of the environment, especially in terms of the statistical distribution of particle displacements in different directions.

Diffusion processes therefore play an important role in the study of the response of dynamic mechanical systems subjected to random excitations, as they can be used for a sufficiently wide class of random processes that model excitation (we speak of random processes with rational spectral densities) and have the advantage that Under certain conditions, general enough for practical applications, their probability densities can be determined by solving differential equations with partial derivatives. In this sense, the following theorem is true.

If $x(t)$ it is a diffusion process and there are continuous partial derivatives

$$\frac{\partial p(x,t|x',t')}{\partial t}, \frac{\partial [a_i(x,t)p(x,t|x',t')]}{\partial x_i} \text{ and } \frac{\partial^2 [b_{ij}(x,t)p(x,t|x',t')]}{\partial x_i \partial x_j}, i, j = 1, 2, \dots, n$$

then for $t' < t$ and $x, x' \in \mathbb{R}^n$, $p(x, t|x', t')$ satisfy the equation

$$\frac{\partial p}{\partial t} = - \sum_{i=1}^n \frac{\partial [a_i(x,t)p]}{\partial x_i} + \frac{1}{2} \sum_{i,j=1}^n \frac{\partial^2 [b_{ij}(x,t)p]}{\partial x_i \partial x_j} \quad (22)$$

With the initial condition

$$\lim_{t \rightarrow t'} p(x, t|x', t') = \delta(x - x') \quad (23)$$

Equation (22) is called the Fokker-Planck equation. If the dissemination process $x(t)$ is a stationary one, then the transition probability density $p(x, t|x')$ satisfies, in its entirety, the equation (22) with initial condition

$$\lim_{t \rightarrow 0} p(x, t|x') = \delta(x - x') \quad (24)$$

A solution $p(x, t|x')$ of the Cauchy problem, in order to be probability density, it is necessary to satisfy the following conditions as well:

$$p(x, t|x') \geq 0, \text{ for any } x, x' \in \mathbb{R}^n \text{ and } t \in \mathbb{R}^+ \quad (25)$$

$$\int_{\mathbb{R}^n} p(x, t|x') = 1, \text{ for any } x, x' \in \mathbb{R}^n \text{ and } t \in \mathbb{R}^+ \quad (26)$$

$$\lim_{t \rightarrow 0} \int_{\mathbb{R}^n} p(x, t|x') \varphi(x') dx' = \varphi(x) \quad (27)$$

for any initial probability density $\varphi(x)$.

In many practical applications, both the transfer vector $a(x, t)$, as well as the diffusion matrix $B(x, t)$ are independent of time. In this sense, if there is asymptotically stable stationary probability density $p_s(x)$, previously defined, then it satisfies, by itself, the homogeneous equation

$$\sum_{i=1}^n \frac{\partial [a_i(x, t) p_s(x)]}{\partial x_i} + \frac{1}{2} \sum_{i,j=1}^n \frac{\partial^2 [b_{ij}(x, t) p_s(x)]}{\partial x_i \partial x_j} = 0 \quad (28)$$

which is called the stationary Fokker-Planck equation.

Assuming that we are dealing with an infinite regular behavior of probability densities $p(x, t|x')$ and $p_s(x)$, the boundary conditions for the above equations, (20, 21), will be respectively

$$\lim_{\|x \rightarrow \infty\|} p(x, t|x') = 0 \text{ and } \lim_{\|x \rightarrow \infty\|} p_s(x) = 0 \quad (29)$$

We also mention that these boundary conditions can be imposed for the vast majority of dynamic systems encountered in daily practice, as they express a physical reality. Because probability densities are positive, it is most often assumed that we have

$$\lim_{\|x \rightarrow \infty\|} \frac{\partial p(x, t|x')}{\partial x_i} = 0 \text{ and } \lim_{\|x \rightarrow \infty\|} \frac{\partial p_s(x)}{\partial x_i} = 0, \quad i = 1, 2, \dots, n \quad (30)$$

3.2.3 Wiener processes

A random process $x(t) \in V_{\mathbb{R}_+}^1$ it is called a process with independent increases if for any finite set $\{t_1, t_2, \dots, t_k | t_i < t_{i+1}\} \subset \mathbb{R}_+$ random variables $x(t_1), x(t_2) - x(t_1), \dots, x(t_k) - x(t_{k-1})$ they are independent. If, in addition, $x(t + \tau) - x(t')$ has the same statistical distribution as $x(t) - x(t')$ for any $t' < t$ and any $\tau > 0$ it can be stated that $x(t)$ it is a process with independent stationary growths.

Let $\varphi(u_1, u_2, \dots, u_k, t_1, t_2, \dots, t_k) = M \left[\exp i \sum_{j=1}^k (u_j \times t_j) \right]$ be characteristic function of the order k of the process $x(t)$ with independent increases. Because

$$\begin{aligned} \sum_{j=1}^k (u_j \times t_j) &= u_k [x(t_k) - x(t_{k-1})] + (u_{k-1} + u_k) [x(t_{k-1}) - x(t_{k-2})] + \dots + \\ &+ (u_2 + \dots + u_k) [x(t_2) - x(t_1)] + (u_1 + u_2 + \dots + u_k) x(t_1) \end{aligned}$$

by virtue of the independence of random variables $x(t_k) - x(t_{k-1})$, $x(t_{k-1}) - x(t_{k-2})$, \dots , $x(t_2) - x(t_1)$, $x(t_1)$, it can be written

$$\begin{aligned} \varphi(u_1, u_2, \dots, u_k, t_1, t_2, \dots, t_k) &= M \left[\left\{ \exp i x(t_1) \sum_{j=1}^k u_j \right\} \left\{ \exp i [x(t_2) - x(t_1)] \sum_{j=2}^k u_j \right\} \times \dots \times \right. \\ &\times \left. \left\{ \exp i [x(t_{k-1}) - x(t_{k-2})] (u_{k-1} + u_k) \right\} \left\{ \exp i [x(t_k) - x(t_{k-1})] u_k \right\} \right] = \\ &= M \left[\left\{ \exp i x(t_1) \sum_{j=1}^k u_j \right\} \right] M \left[\left\{ \exp i [x(t_2) - x(t_1)] \sum_{j=2}^k u_j \right\} \right] \times \dots \times M \left[\left\{ \exp i [x(t_k) - x(t_{k-1})] u_k \right\} \right] \end{aligned}$$

hence it follows that in order to fully characterize a process with independent growths, it is sufficient to give its distribution $x(t)$ and $x(t) - x(\tau)$ for any $\tau < t \in T$.

A random process $w(t) \in V_{\mathbb{R}_+}^1$ is called the Wiener process if:

- i. $w(t)$ it is a process with independent stationary growths;
- ii. $w(t)$ has a normal distribution;
- iii. $M[w(t)] = 0$;
- iv. $P(\{\omega \in \Omega | w(\omega, 0) = 0\}) = 1$.

Wiener processes are also called Brownian processes because they were used to model the motion of a particle suspended in a liquid, a motion caused by its collision with the molecules of the liquid. To determine the analytical expression of the quadratic mean $\sigma_w^2(t)$ of a Wiener process $w(t)$ we consider the relation

$$\sigma_w^2(t + \tau) = M[w^2(t + \tau)] = \dots = M[w^2(t)] + M[\{w(t + \tau) - w(t)\}^2] \quad (31)$$

valid by virtue of the independence of growth $w(t + \tau) - w(t)$ and $w(t) - w(0) = w(t)$ and the fact that $M[w(t)] = 0$. On the other hand, the stagnation of the process $w(t)$ increases involve

$$M\left[\{w(t+\tau)-w(t)\}^2\right]=\dots=M\left[w^2(t)\right] \quad (32)$$

as such

$$\sigma_w^2(t+\tau)=\sigma_w^2(t)+\sigma_w^2(\tau) \quad (33)$$

which is a functional equation whose solution $\sigma_w^2(t)$ is in form $\sigma_w^2(t)=ct$, where c it is a positive constant, which in practical cases can be determined experimentally.

Since a Wiener process is a normal process with zero mean and square mean of form (33), its first-order probability density has the expression

$$p(w,t)=\frac{1}{\sqrt{2\pi ct}}\exp\left(\frac{w^2}{-2ct}\right) \quad (34)$$

To determine the correlation function $R_w(t,t')$ of the Wiener process $w(t)$ we write

$$\begin{aligned} R_w(t,t') &= M[w(t),w(t')] = M[\{w(t)-w(t')+w(t')\},w(t')] = \\ &= M[\{w(t)-w(t')\}]M[w(t')] + M[w^2(t')] = \sigma_w^2(t') = ct' \end{aligned} \quad (35)$$

for $t \geq t'$; analogue, $R_w(t,t') = ct$ for $t \leq t'$. As such, $R_w(t,t') = c \min(t,t')$.

Because the correlation function $R_w(t,t')$ it is continuous at any point $(t,t') \in \mathbb{R}_+ \times \mathbb{R}_+$, it turns out that the random process, and at the same time Wiener, $w(t)$, is continuous on the square average on \mathbb{R}_+ and is therefore also integrable on a square average over any interval $T \subset \mathbb{R}_+$ considered.

On the other hand $\frac{\partial^2 R_w(t,t')}{\partial t \partial t'}$ it does not exist at any point $(t,t') \in \mathbb{R}_+ \times \mathbb{R}_+$ and so Wiener processes are not derivable anywhere in the quadratic mean. Moreover, they are not even with a limited variation in the square mean. A Wiener process $w(t)$ is a Markov process, since

$$\begin{aligned} & p(w_k, t_k | w_1, w_2, \dots, w_{k-1}, t_1, t_2, \dots, t_{k-1}) = \\ & = p(w_k - w_{k-1} + w_{k-1} - w(0), t_k | w_1 - w(0), w_2 - w(0), \dots, w_{k-1} - w(0), t_1, t_2, \dots, t_{k-1}) = \\ & = p(w_k - w_{k-1} + w_{k-1} - w(0), t_k | w_{k-1} - w(0), t_{k-1}) \\ & = p(w_k, t_k | w_{k-1}, t_{k-1}) \end{aligned} \quad (36)$$

3.2.3 White noise

White noise is, in turn, a widely used random process in modeling dynamic mechanical systems subjected to random excitations. Although this process is not physically feasible (its energy being infinite), it can still

become a useful approximation for many other random processes encountered in practice, especially when subjected to filtering operations. Using white noise, a series of simplifications are obtained with importance for the study of stochastic differential equations. White noise can also be formally defined in different ways.

A Markov process $z(t) \in V_{\mathbb{R}_+}^1$ it's called white noise if for any $t > t'$ we have that

$$p(z, t | z', t') = p(z, t), \text{ pentru } z, z' \in \mathbb{R}. \quad (37)$$

The above relationship expresses that for any $t, t' \in \mathbb{R}_+$, $z(t)$ and $z(t')$ are independent random variables or, in other words, the values it can take $z(t)$ at two different time points they do not statistically depend on each other, no matter how close the two time moments are. This is of course physically impossible. Although the above statements express a number of properties specific to white noise, they are not useful in practical applications; because of this, white noise can be more easily characterized indirectly. Thus, in the following, we will consider that $z(t)$ it is a stationary random process with normal distribution.

Let $x^\rho(t)$ be a normal stationary random process with zero mean having the correlation

$$\text{function } R_x^\rho(\tau) = 2\pi S_0 \frac{\rho}{2} e^{-\rho|\tau|}, \text{ pentru } \rho > 0 \text{ și } S_0 > 0 \quad (38)$$

For sufficiently large values of ρ properties of the random process $x^\rho(t)$ approximates those of a normal stationary white noise, as the correlation function $R_x^\rho(\tau)$, which expresses the statistical dependence between $x^\rho(t)$ and $x^\rho(t + \tau)$, takes very small values even for values of τ close to zero. If, instead, ρ is an integer then the sequence of functions $R_x^{\rho_n}(\tau)/2\pi S_0$, where $\rho_1 < \rho_2 < \dots$, generates the Dirac distribution $\delta(\tau)$. Under these new conditions, we are thus led to the following definition of white noise.

A stationary random process with normal distribution $z(t)$ is called normal white noise if

- i. $M[z(t)] = 0$;
- ii. $R_z(\tau) = M[z(t)z(t + \tau)] = 2\pi S_0 \delta(\tau)$.

Because $\delta(\tau)$ it is not an ordinary function, but rather a generalized function, it is seen once again that white noise is a purely mathematical concept. Also, the spectral density of the random process $x^\rho(t)$ having the correlation function (38) has the expression

$$S_x^\rho(\nu) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_x^\rho(\tau) e^{-i\nu\tau} = \frac{S_0}{1 + \left(\frac{\nu}{\tau}\right)^2} \quad (39)$$

For $\rho \rightarrow \infty$ we obtain that $S_z(\nu) \equiv S_0$, property that underlies the name of white noise of the random process $z(t)$, by analogy with white light, which contains all the frequency components. The fact that the

spectral density of white noise is constant over the entire frequency range, implies infinite energy of it, which gives it the character of the unfeasible physique. If the spectral density of a random process is approx. constant to sufficiently high frequencies, this is called a broadband random process, in which case white noise can be a useful approximation.

Normal stationary white noise can also be defined from Wiener processes, this definition is considered in the case of Itô-type stochastic differential equations. Since the random process $z(t)$ is not derivable on a quadratic average, the distribution defined by the relation is considered

$$\langle \varphi, \theta_i \rangle = \int_{-\infty}^{\infty} \varphi(t') \theta_i(t') dt' = \int_{-\infty}^{\infty} \varphi(t') dt' \quad (40)$$

which is derived from the distribution

$$\langle \varphi, \theta_i' \rangle = - \langle \varphi', \theta_i \rangle = - \int_{-\infty}^{\infty} \varphi(t') dt' = \varphi(t) = \langle \varphi, \delta_i \rangle \quad (41)$$

Therefore, in the sense of the theory of distributions, the relationship can be written immediately

$$\frac{\partial^2 R_w(t, t')}{\partial t \partial t'} = \delta(t - t') \quad (42)$$

which can be seen as an extension of the necessary and sufficient condition of derivability in the quadratic mean of random processes and therefore only in this sense can normal stationary white noise be defined as the derivative of a Wiener process. Next, we will specify this definition by notation

$$dw(t) = z(t) dt \quad (43)$$

where $dw(t) = w(t + dt) - w(t)$ represents the random process of infinitesimal increases of the Wiener process $w(t)$ and has the properties

- i. $M[dw(t)] = 0$;
- ii. $M[\{dw(t)\}^2] = c dt$;
- iii. $M[dw(t) dw(t')] = 0$, for $t \neq t'$.

The following is a process for generating white noise as the limit of a series of random processes whose achievements take constant values over time Δt , then $\Delta t \rightarrow 0$. Let $z^\Delta(t) \in V_{\mathbb{R}_+}^1$ be the random process defined by the relation

$$z^\Delta(t) = z_k, \quad k\Delta t \leq t < (k+1)\Delta t, \quad k = 0, 1, 2, \dots \quad (44)$$

where z_k are normal random variables with zero mean and independent two by two, so that

$$M[z_l z_m] = c\delta_{lm}, \quad l, m = 0, 1, 2, \dots \quad (45)$$

c being in this case a positive constant. Whether $T \in \mathbb{R}_+$, $T/\Delta t$ is an integer and we consider the variable random $y^\Delta(t)$ whose values are obtained by integrating the achievements of the random process $z^\Delta(t)$ on the interval $[0, T]$, so $y^\Delta(t) = \int_0^T z^\Delta(t) dt$, where obviously $M[y^\Delta(T)] = 0$. From the above relations we obtain that

$$\begin{aligned} M\left[\left\{y^\Delta(T)\right\}^2\right] &= M\left[\int_0^T z^\Delta(t) dt \int_0^T z^\Delta(t') dt'\right] = \dots = \\ &= \int_0^T \int_0^T M\left[z^\Delta(t) z^\Delta(t')\right] dt dt' = \sum_{k=0}^{T/\Delta t} \int_{k\Delta t}^{(k+1)\Delta t} \int_{k\Delta t}^{(k+1)\Delta t} c dt dt' = Tc\Delta t. \end{aligned}$$

It is observed that if c it is kept constant then $M\left[\left\{y^\Delta(T)\right\}^2\right] = 0$ when $\Delta t \rightarrow 0$. This is equivalent to the effect of random excitation $z^\Delta(t)$ in the differential equation $\frac{dy^\Delta(t)}{dt} = z^\Delta(t)$ is reduced to zero for $\Delta t \rightarrow 0$, which is physically unacceptable. Therefore to keep the mean square $M\left[\left\{y^\Delta(T)\right\}^2\right]$ nonzero constant when $\Delta t \rightarrow 0$, c must be replaced with $c/\Delta t$. This means that the amplitude of the steps in the achievements of the random process $z^\Delta(t)$ grows like $1/\sqrt{\Delta t}$, while the area below these tends to zero as $\sqrt{\Delta t}$. It is thus highlighted that $z^\Delta(t)$ will have the limit for $\Delta t \rightarrow 0$ the white noise properties.

The above-described white noise generation method can be used for computer numerical simulation of the achievements of a random process that satisfactorily approximates the achievements of the white noise. To this end, it is necessary to obtain a set of independent normal random variables with zero mean and square mean $2\pi S_0 / \Delta t$, where $S_z(\nu) \equiv S_0$ is the spectral density of white noise, and Δt it is the sampling step of his achievements. The strings of values (the realizations themselves) corresponding to such a string of random variables are determined using random numbers. Experience has shown that the congruence method is best suited for generating random numbers on a computer.

According to this method, a string of random numbers v_k , $k = 1, 2, \dots$ it is given by the iterative process $v_{k+1} = av_k + b \pmod{P}$, where $v_{k+1} \in (0, P]$ with $k = 1, 2, \dots$, in which b and P are prime numbers. The choice of P it is done according to the capacity and numbering base of the computer. The constants are chosen so that the correlation between the values obtained is as small as possible, the period of the string of random numbers is as long as possible and the speed of generation of random numbers is respectively. Thus, approximate numerical representations of the achievements of a normal stationary white noise are obtained, the accuracy of the representation depending on the sampling step Δt .

3.3 Study of dynamic systems behavior through automatic generation of random processes

In the study undertaken on linear dynamical systems, we considered important elements that give particularity to random processes, among which we mention the distribution function, probability density function, statistical moments of a random process, time averages, and correlation (and intercorrelation) of a random signal (process).

Microsoft Excel® offers over 50 statistical functions that are useful in everyday practice. Some of these are available immediately after installing Excel, others, the least used, can be installed on-demand using Analysis ToolPak® (Fig. 1). However, before using an analysis tool, the data to be analyzed must be sorted and arranged in rows or columns in the spreadsheet. These will be the input data domain. If the Data Analysis command is not in the Tools menu then the Analysis ToolPak interface must be installed in Microsoft Excel.

To install Analysis ToolPak, follow these steps: On the Tools menu, click Add-Ins. If Analysis ToolPak is not listed in the Add-Ins dialog box, click the Browse button and locate the disk drive, folder, and file name for the Analysis ToolPak add-in package, called Analys32.xll. It is usually located in the Microsoft Office \ Library \ Analysis folder. If it is not installed then run the MS Excel installer again. Select the Analysis ToolPak checkbox. To use Analysis ToolPak, select Data Analysis from the Tools menu. In the Analysis Tools dialog box, select the tool that you want to use. Enter the input and output range then select the desired option.

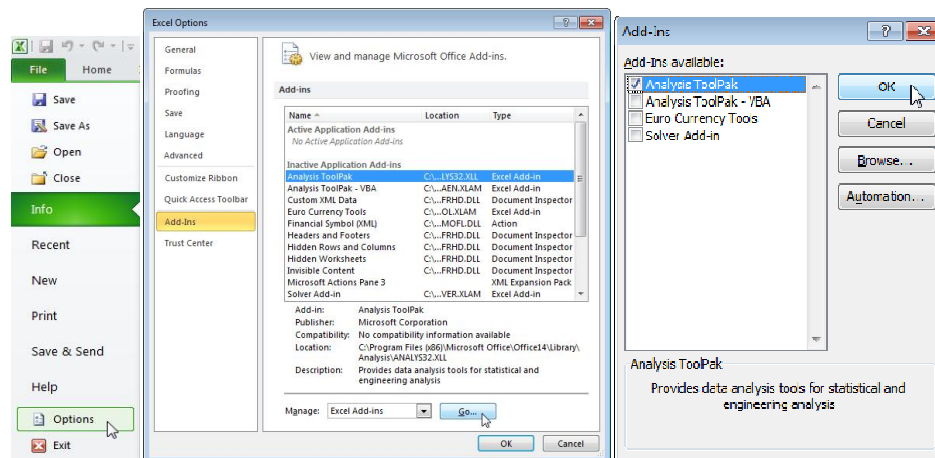


Fig. 1. Analysis toolpak in Microsoft excel®

Below are some of the most used statistical functions. Both the statistical functions of the application and the procedures obtained through Tools - Data Analysis can be used to process a data set stored in an Excel document. The usual statistical functions are (in alphabetical order):

AVEDEV - absolute mean deviation	MIN, MAX - extreme values in the list
AVERAGE - arithmetic mean	MEDIAN – median
BINOMDIST - binomial distribution function	NORMDIST - normal distribution function
CHIDIST - distribution function χ^2	STDEV - standard deviation
HARMEAN - harmonic average	VAR - dispersion

To use statistical procedures, you must verify with Tools - AddIns that the Analysis ToolPak utility is installed. If so, the Tools - Data Analysis command will open the Data Analysis dialog from which a series of statistical processing driven by the associated dialogs are accessible. Thus, Descriptive Statistics will

produce the statistical indicators of a continuous variable. Also in the statistical description part can be included the method of creating OLAP cubes through Data - Pivot Table, a method by which simple or multivariate distributions of discrete variables or essential statistical indicators of subpopulations are obtained. Also, we must not forget that all the procedures related to graphical representations, initiated by Insert - Chart are a part of the descriptive statistical processing offering graphs, histograms, etc. The following are several procedures available through the Tools - Data Analysis dialog.

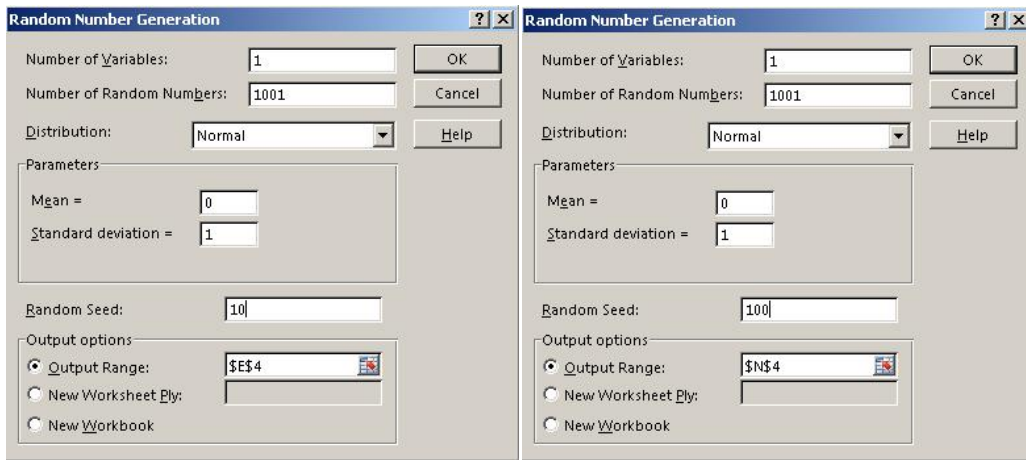
The Random Number Generation analysis tool fills a range with independent random numbers derived from one of several distributions. You can characterize the subjects of a population with a probability distribution. For example, a normal distribution can be used to characterize the population of individuals' heights, or a Bernoulli distribution of two possible consequences can be used to characterize the population of the money experiment results.

Using this procedure, random number series can be generated distributed according to 7 different types of distribution functions. The result consists of one or more columns of numbers, each column representing values of a variable distributed according to a specified distribution function. For each generation, the number of columns (variables) generated, the number of values (the same for all variables), the type of the distribution function, the parameters of the function, and the place where the results will be entered will be given.

To determine the normal distribution it is necessary to specify the values for the mean and the standard deviation of the population. Mean - specify the value for the population average. Standard Deviation - specify the value for the standard deviation of the population. The default values are those of the standard normal distribution, the mean 0 and the standard deviation 1. In the following, for the beginning, it is presented, by way of example, the generation by a normal distribution, with a given mean and standard deviation, of some random data sets, using Data Analysis - Random Number Generation module.

3.3.1 Case no. 1 - mean (0) and standard deviation (1)

For the first case we will consider an average (mean) equal to 0 and a standard deviation equal to 1. At the same time for the data set - random variables (see Fig. 2), we will propose an analysis by expressing correlation and covariance (see Fig. 3).



Parameters	random seed time (t)	10	20	30	40	50	60	70	80	90	100	
		$x_1(t)$	$x_2(t)$	$x_3(t)$	$x_4(t)$	$x_5(t)$	$x_6(t)$	$x_7(t)$	$x_8(t)$	$x_9(t)$	$x_{10}(t)$	
mean	0	0	-2.85283	-2.73241	-2.63957	-2.56507	-2.50431	-2.45007	-2.40219	-2.3605	-2.32141	-2.28558
st. deviation	1	0.01	0.03944	0.82704	-1.43235	-0.36866	0.3487	1.38192	-0.85361	-0.05813	0.69663	-1.78525
		0.02	-1.28237	0.12961	-2.71986	-0.11304	1.32073	-0.36271	0.87843	-0.63766	0.5638	-0.97308
		0.03	-0.20722	0.71103	-1.25366	-0.12937	0.81365	-1.10033	-0.05223	0.92577	-0.96915	0.02452
		0.04	-0.20698	0.16557	0.5637	1.08615	-2.36425	-1.00781	-0.51137	-0.11981	0.25349	0.66766
		0.05	0.14683	0.69575	1.69621	-1.02202	-0.38067	0.1259	0.66958	1.61436	-1.05762	-0.40305
		0.06	-0.82338	-1.01533	-1.2545	-1.60127	-2.6108	1.68884	1.30605	1.05401	0.8557	0.68633
		0.07	-1.32901	-0.84658	-0.50928	-0.22256	0.04672	0.31941	0.61874	0.98879	1.60099	-1.6263
		0.08	-0.36418	-0.26163	-0.16177	-0.0635	0.03416	0.13207	0.23135	0.33297	0.43815	0.54944
		0.09	-1.57218	-0.90876	-0.50867	-0.1783	0.13331	0.45871	0.84352	1.43471	-1.6642	-0.94712
		0.1	-1.11005	-1.35098	-1.71454	2.89561	1.67437	1.32753	1.08252	0.90611	0.74685	0.60453
		0.11	1.13127	-1.19757	1.26976	1.34908	1.43814	1.54009	1.68146	1.81343	2.02519	2.40911
		0.12	0.81035	-0.11984	-0.51399	-1.55606	0.9007	0.18242	-0.44372	-1.37322	0.99908	0.24654
		0.13	-1.57986	-1.23918	-1.00603	-0.81781	-0.68491	-0.50378	-0.3709	-0.24071	-0.1145	0.00991
		0.14	-0.25523	-0.94724	1.59197	0.57342	-0.02662	-0.63775	-1.62101	0.66834	0.20027	-0.38034
		0.15	0.75231	-1.48927	-0.36018	0.38676	1.56929	-0.72947	0.06028	0.89726	-1.24814	-0.25974
		0.16	-0.1075	0.36214	0.93687	-2.33522	-0.86334	-0.31025	0.15704	0.66384	1.48189	-1.20039
		0.17	-0.18889	1.66461	0.44262	-0.54205	1.38411	0.0998	-0.98345	0.79138	-0.23127	-1.85969
		0.18	0.38577	-0.81046	0.73077	-0.4504	1.20008	-0.14181	-2.83537	0.15348	-1.17667	0.48322
		0.19	0.32433	0.86445	2.18359	-0.97566	-0.40214	0.0576	0.53056	1.18142	-1.5512	-0.70759
		0.2	-1.15583	0.27313	-1.33254	0.18935	-1.56485	0.10688	-1.93858	0.02513	2.49907	-0.05845
		0.21	0.51766	-0.1966	-1.05174	1.13054	0.24118	-0.46859	-1.70498	0.73397	-0.01779	-0.78127
		0.22	-0.69663	0.93544	-0.23434	2.30913	0.18071	-1.02009	0.63111	-0.47286	1.28359	-0.04389
		0.23	1.48556	-1.46133	-0.79725	-0.37631	-0.01496	0.34439	0.7568	1.37934	-1.58253	-0.85075
		0.24	-2.05903	-0.05959	1.49649	-0.28148	0.01803	-0.51828	0.7019	-0.78897	0.44422	-1.13578
		0.25	-0.21402	0.80685	-0.97418	0.10011	1.37068	-0.55449	0.42472	-1.75842	-0.21668	0.80336

Fig. 2. Random variables for 1st case - mean (0) and standard deviation (1)

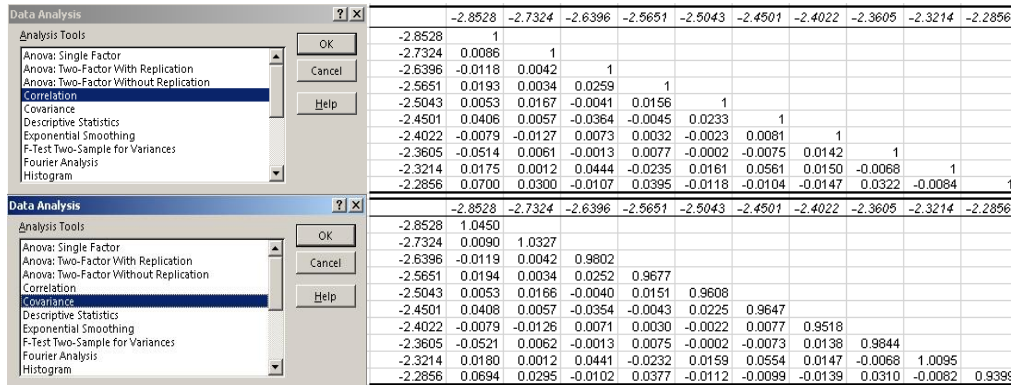
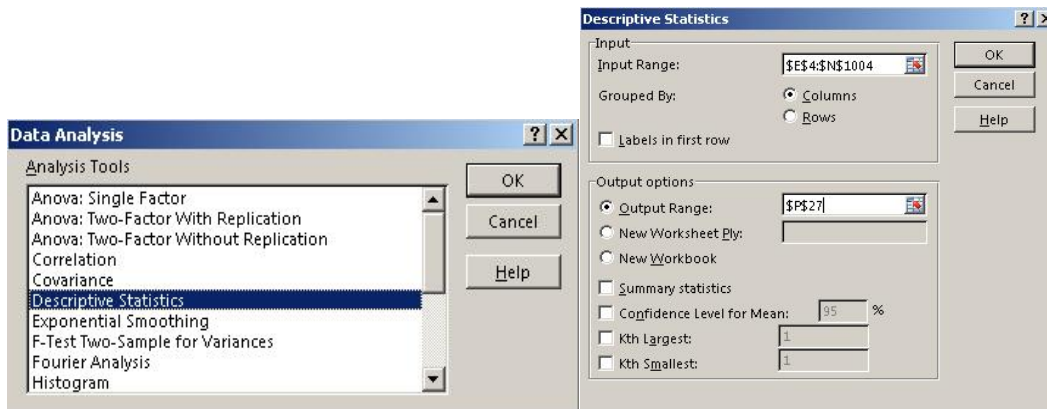


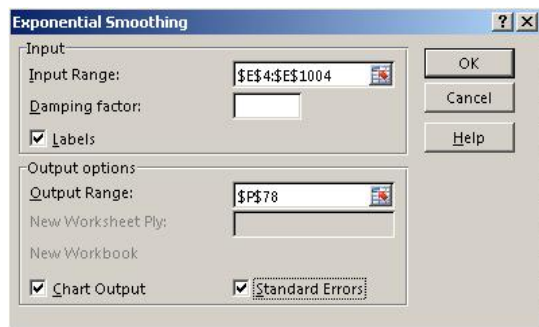
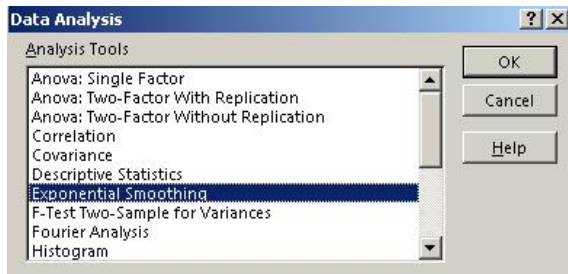
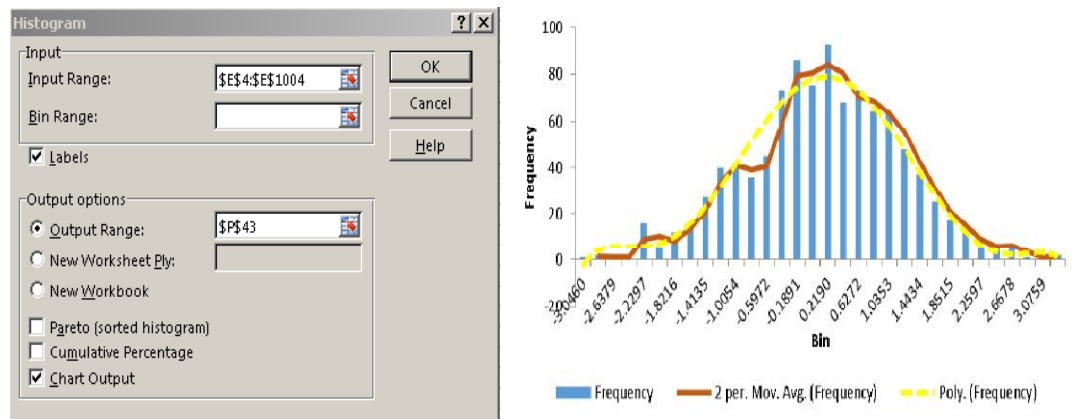
Fig. 3. Correlation and covariance of random variables for 1st case

Respectively by exposing the descriptive statistics (see Fig. 4), the histogram and the related exponential smoothing (see Fig. 5).



	$x_1(t)$	$x_2(t)$	$x_3(t)$	$x_4(t)$	$x_5(t)$	$x_6(t)$	$x_7(t)$	$x_8(t)$	$x_9(t)$	$x_{10}(t)$
	-2.8528	-2.7324	-2.6396	-2.5651	-2.5043	-2.4501	-2.4022	-2.3605	-2.3214	-2.2856
Mean	0.04678	-0.0423	-0.0672	0.02098	-0.0116	0.01632	-0.0206	0.04155	0.02523	-0.0601
Standard Error	0.03234	0.03215	0.03132	0.03112	0.03101	0.03108	0.03087	0.03139	0.03179	0.03067
Median	0.06334	-0.0583	-0.0833	0.00298	-0.0158	0.05269	-0.0142	0.02961	0.06952	-0.0726
Mode	0.73808	0.69575	-2.7199	-0.9981	1.67437	-0.6377	0.82145	-0.1905	0.54214	-0.3902
Standard Deviation	1.02277	1.01671	0.99056	0.98421	0.98069	0.9827	0.97611	0.99268	1.00525	0.96997
Sample Variance	1.04606	1.03371	0.98122	0.96866	0.96175	0.96569	0.9528	0.98542	1.01052	0.94083
Kurtosis	-0.0108	0.05151	-0.0444	-0.2903	0.20504	-0.0606	-0.2482	-0.1489	-0.3353	0.00053
Skewness	-0.072	-0.0523	-0.0098	0.10834	-0.0759	-0.0267	-0.0267	-0.0681	-0.1133	0.17731
Range	6.32601	6.833	6.33096	5.39076	6.89994	6.43078	5.86595	6.83794	5.74131	6.3538
Minimum	-3.046	-3.4558	-3.6682	-2.3821	-3.5227	-3.4558	-2.9948	-3.3152	-3.1067	-2.8311
Maximum	3.28	3.37721	2.66271	3.00861	3.37721	2.97499	2.87117	3.52273	2.63462	3.52273
Sum	46.7815	-42.255	-67.178	20.9776	-11.619	16.3186	-20.591	41.5499	25.2257	-60.053
Count	1000	1000	1000	1000	1000	1000	1000	1000	1000	1000

Fig. 4. The descriptive statistics of random variables for 1st case



#N/A	#N/A
0.039444785	#N/A
-0.885825102	#N/A
-0.41079943	#N/A
-0.268126996	0.86588
0.022342078	0.47407
-0.569664405	0.55647
-1.101208264	0.69858
-0.58529157	0.7821
-1.27611335	0.83542
-1.159871945	0.71757
0.443926131	1.44347
0.70042491	1.34303
-0.889475268	1.8746
-0.445502242	1.37783
0.399965558	1.52969
0.044742851	0.84036
-0.118795051	0.76831
0.234399784	0.43462
0.297349594	0.32519
-0.719878653	0.88965
0.146399557	1.10323
-0.443717768	1.2047
0.906779538	1.41

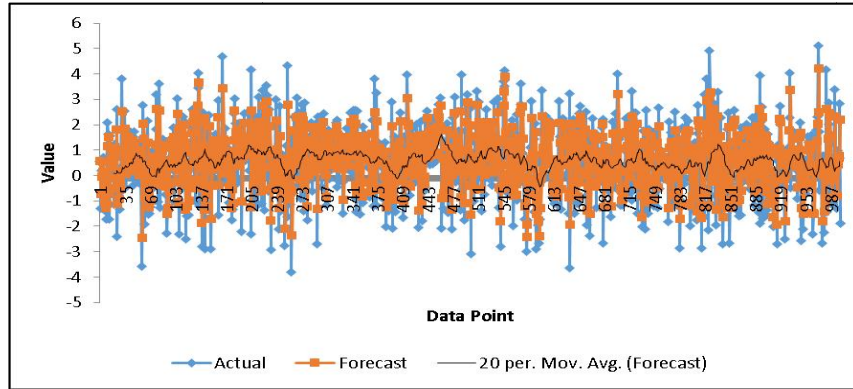


Fig. 5. Histogram and related exponential smoothing of random variables for 1st case

3.3.2 Case no. 2 - mean (0.5) and standard deviation (1.41)

For the second case we will consider a mean equal to 0.5 and a standard deviation equal to 1.41. At the same time for the data set - random variables (see Fig. 6), we will propose an analysis by expressing correlation and covariance (see Fig. 7).

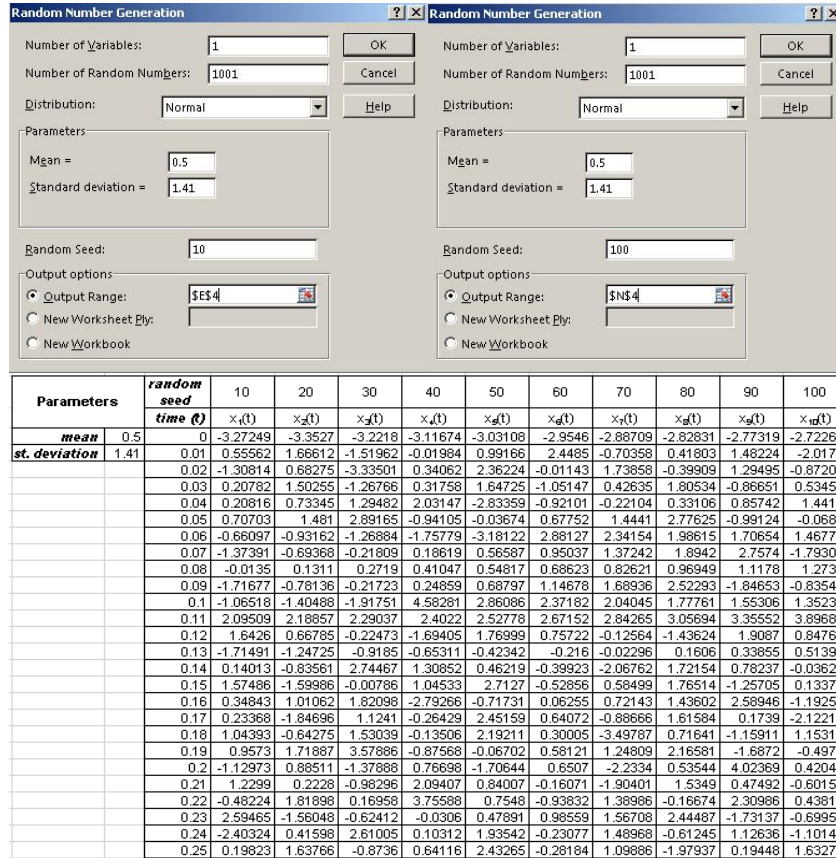


Fig. 6. Random variables for 2nd case - mean (0.5) and standard deviation (1.41)

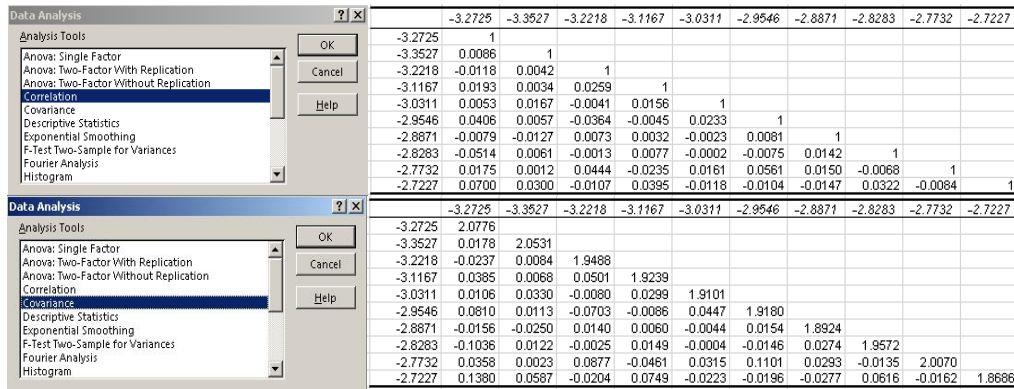


Fig. 7. Correlation and covariance of random variables for 2nd case

Respectively by exposing the descriptive statistics (see Fig. 8), the histogram and the related exponential smoothing (see Fig. 9).

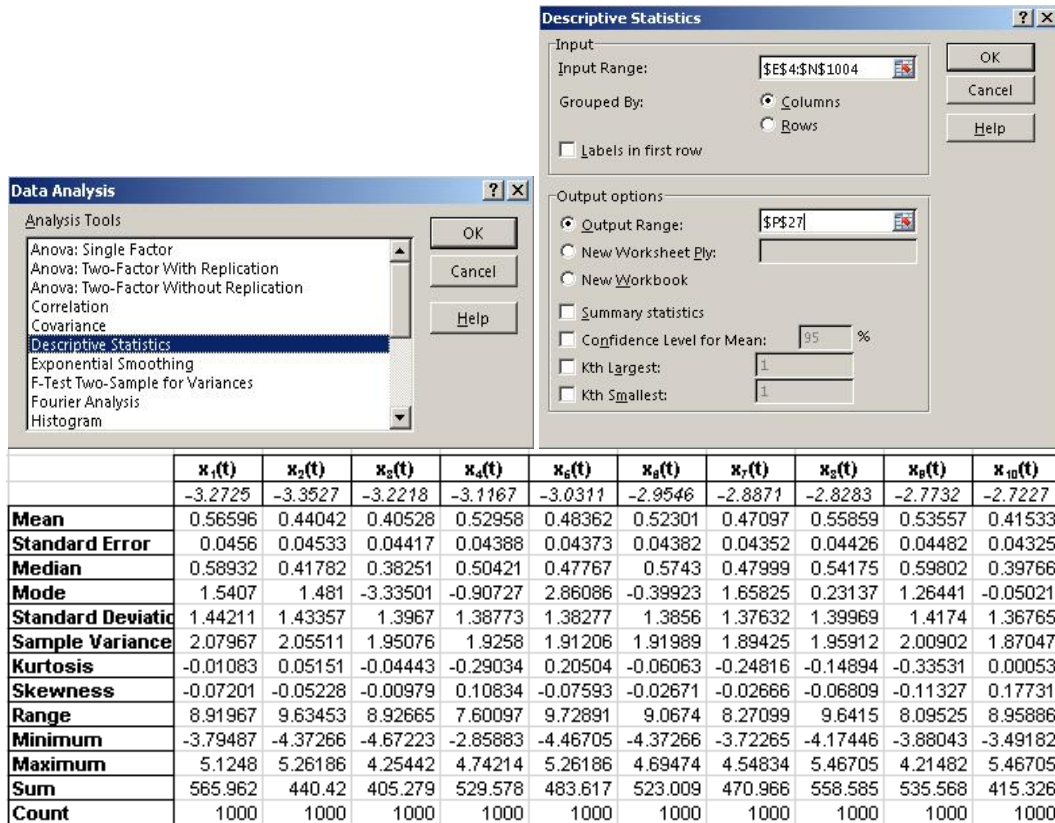


Fig. 8. The descriptive statistics of random variables for 2nd case

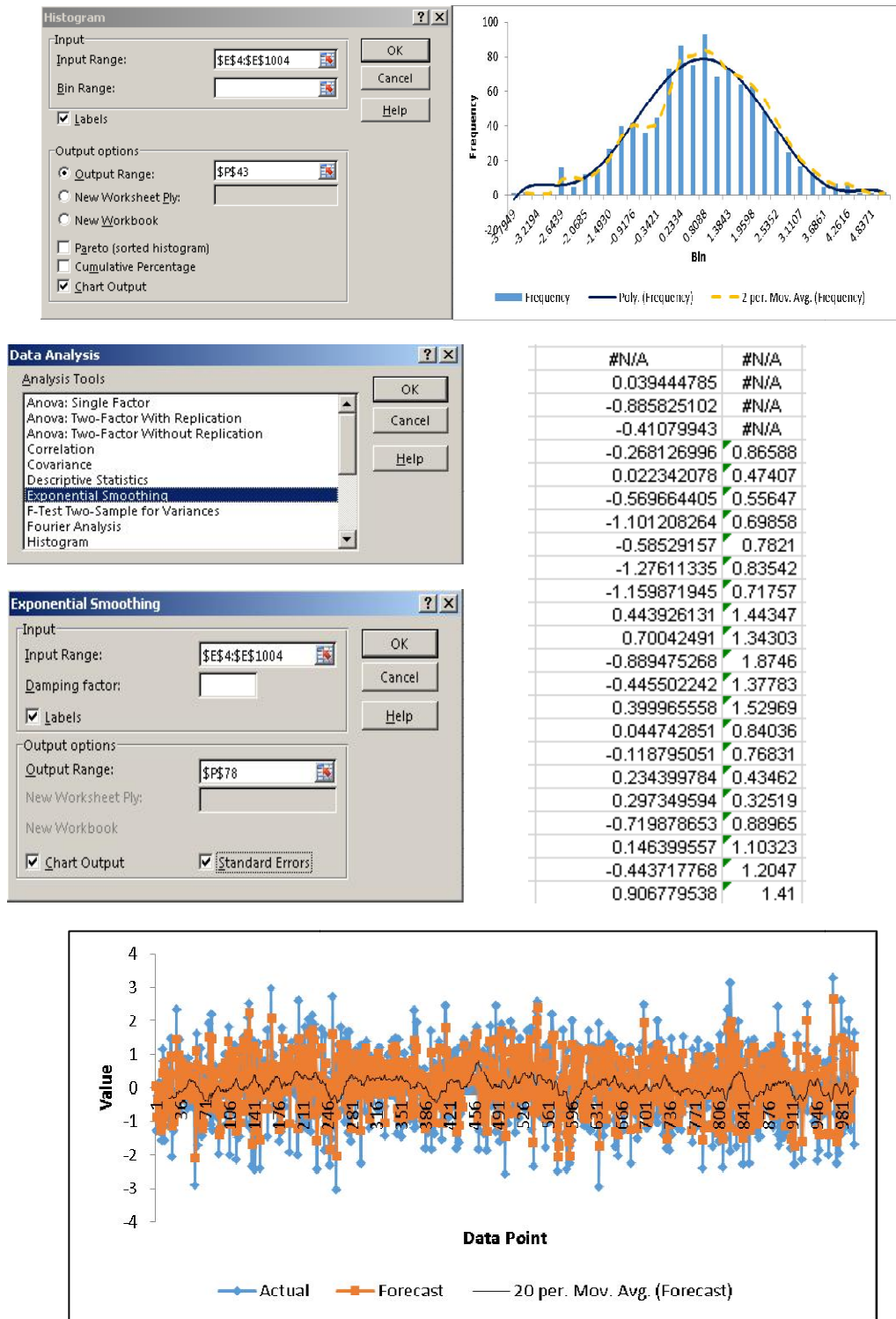


Fig. 9. Histogram and related exponential smoothing of random variables for 2nd case

4 Conclusions

Through this paper, we have tried to bring to the fore the implications that random processes have on the study of the dynamic system. We started from the presentation of the basic concepts with which we are accustomed to probability and statistics, getting to study the Markov, diffusion, Wiener and white noise processes. The multitude of equations come to certify the preoccupation for the staging of the dynamic systems, the transition from one process to another being done gradually, with the consideration and maintenance of specific notations. The implications of random processes in the study of dynamical systems are reviewed, as well as a series of applications specific to the analysis of dynamic behavior.

Competing Interests

Author has declared that no competing interests exist.

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