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Enhanced Technique of Constructing Multiple ODD Magic Square Matrices

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

An enhanced technique of constructing multiple odd magic square matrices is proposed in this work. A specific rule of establishing improved odd magic to magic squares derived from the odd algebraic Latin squares is studied and programmed here. Magic squares are practically important from the properties of their equality in the sum of rows, columns and diagonals. An $n \times n$ odd magic square is an array containing the positive integers from 1 to n^2, arranged so that each of the rows, columns, and the two principal diagonals have the same sum.

Keywords: Latin squares; magic squares; pivot element; vertical and horizontal pivot elements; OMS; elliptic curves.

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1 Introduction

A magic square (MS) is a square array of numbers in which each row, column or diagonal of numbers has the same sum. Adding the elements of each row, each column or each diagonal of numbers in a magic square will give the same sum. This same sum is called the magic constant. Further, an odd magic square (OMS) isa square array of odd dimension (i.e. n by n array where n is an odd integer), which is a magic square too. As such in the example given below, 15 is the magic number. Now, one can work this out just by knowing that the square uses the natural numbers (1, 2, 3,…) from 1 to 9, where $n = 3$ [1].

8	1	6
3	5	7
4	9	2

Fig. 1. First Normal 3×3 Magic Square

Also, the two numbers that are opposite each other across the centre number will add up to the same number [2]. That means in the square, 8 + 2 = 10, 6 + 4 = 10, 1 + 9 = 10 and 3 + 7 = 10. This is because 5 is the pivot element.

The order of a square array (SA) of numbers tells us how many rows or columns the array has. So, a square array of distinct numbers (here, we considered consecutive integers only) with 3 rows and columns is of order 3, and a square with 4 rows and columns is of order 4 and so on. The numbers in the magic square are special. It seems that from ancient times they were believed to be connected with the supernatural and magical world. The earliest record of magic squares is from China about 2200 BC [3,4]. There are number of magic squares, it may be even or odd or doubly even. But in our work, an attempt has been made odd magic squares (OMS). Once an OMS is constructed from odd SA, then we can search for a number of odd magic squares that can be constructed until all OMSs of same order are constructed using a suitable algorithm.

1.2 Related works

The following related papers were briefly forecast and discussed herewith.

(i) The approach of 'Add-on Security Model for Public-Key Cryptosystem Based on Magic Square Implementation' was first introduced by Ganapathy G, and Mani K. [5]. Thismodel will increase the security due to its complexity in encryption because it deals with the magic square formation with seed number, start number and sum that cannot be easily traced. The encryption/decryption is based on numerals generated by magic square rather than ASCII values. This paper provides another layer of security to any public key algorithms such as RSA, ElGamal, etc. Since, this model is acting as a wrapper to a public key algorithm; it ensures that the security is enhanced. Further, this approach is experimented in a simulated environment with 2, 4, 8 and 16 processor model using Maui scheduler which is based on back filling philosophy.

(ii) Ezra (Bud) Brown observed the results that ties many mathematical threads together, threads that originate in several different areas of mathematics [6]. The result is that every elliptic curve has nine points of inflection which can be arranged in a very natural way, to form 3×3 magic squares. The first encounter at the place where arithmetic meets geometry was undoubtedly the 3×3 magic square, an arrangement of the numbers 1 through 9 in a 3×3 square grid so that the numbers in each line of three – that is, each row,each column and two main diagonals – add up to 15 as in Fig. 1. This square matrix is full of surprise, including the fact [7] that $816^2 + 357^2 + 492^2 = 618^2 + 753^2 + 294^2$. The sum of the squares of the three rows (8 1 6), (3 5 7) and (4 9 2) is equal to the sum of the three inverse rows (6 1 8), (7 5 3) and (2 9 4) of the first magic square. And we can see many more magic square musings [8]. Partly due to their connection with Fermat's Last Theorem [9], elliptic curves become an interest of study now a days.

(iii) We briefly recall the paper by Tomba I. entitled as 'A Technique for Constructing Odd-order Magic Squares Using Basic Latin Squares' in the International Journal of Scientific and Research Publications, Vol. 2, Issue 5 in 2012. This technique could be utilised for finding magic squares from basic Latin Squares for any order $n \geq 1$, where n is odd) [10]. However, this method can't be applied to find the even ordered magic squares.

(iv) Salam S. & Moirangthem S. published the paper entitled 'Construction of Multiple Odd Magic Squares' Asian Journal of Mathematics & Computer Research, 29(1): pp. 42-52, May, [2022]. The method implied in the paper could improve the technique applied by Tomba I. for generating n-2 number of odd magic squares from n odd ordered magic squares [11]. The authors could also acquire a technique of constructing multiple odd magic square matrices and programmed that.

(v) Salam S, Longjam J. and Moirangthem S. published a paper entitled 'Encryption Technique of Concealing Highly Explosive Chemicals with Multiple Odd Magic Square Constructions', in the Quest Journal of Research in Applied Mathematics, Vol. 9, Issue-3; pp 22-31, (2023). This paper details the technical aspects of the RSA (Rivest Shamir Adleman) – Cryptosystems in accordance with some more complicated case of Odd Magic to Magic Square constructions. It will find complexity to trace out the pivot element and elements to be filled up in the remaining cells in such magic square matrices [9]. This will give the hackers a headache to decrypt the original message.

2 Preliminaries

The proposed transformation of an SA to OMSs involves the following steps.

(i) Construction of Basic Latin square (BLS) in which sums of elements of each column are the same [11]
(ii) Choose the first column of the BLS and fill up the middle or pivot column of the new matrix
(iii) Fill up the remaining elements accordingly as vertical or horizontal.

2.1 Basic latin square

Let us consider a 3×3 odd Latin square with the elements of $a_{11}, a_{12}, \cdots, a_{33}$ [12].

Representing the above in algebraic form of Latin Square, we have

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{22} & a_{23} & a_{21} \\ a_{33} & a_{31} & a_{32} \end{bmatrix} \quad \text{which can be written as} \quad \begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 4 \\ 9 & 7 & 8 \end{bmatrix}$$

Fig. 2(a). Algebraic form of Latin Square **Fig. 2(b). Latin Square 3×3**

In all cases, the Latin letters are seen once in each row and column. Here, the sums of all columns are equal but are not equal to the sum of the diagonals i.e. $\sum_i d_{ij} \neq \sum_j d_{ij}$, where $a_{11} \neq a_{21} \neq \cdots \neq a_{33}$ and so on. Then the ultimate normal magic square of 3×3 as in Fig. 1 is

$$\begin{bmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix}$$

Fig. 2(c). Normal Magic Square 3×3

The only thing we must remember is to imagine that the matrix has a wrap-around process while the elements of a column/row is reassigned as element of columns/rows transformed matrix, i.e., if we move off one edge of the magic square, we re-enter on the other side. Thus, in Fig. 2(c), from the number 1 we move up or right (with wrap-around) to the bottom right corner to place a 2. Then we move again (with wrap-around) to the middle left to insert the 3. Then we cannot move up or right from here, so we move down to the bottom left, and place the 4. If we continue in this manner, we can acquire the first odd magic square as in Fig. 2(c) of order 3×3. It's that simple. By doing so we will ensure that every square gets filled [13].

2.2 Construction of Odd Magic Squares (OMS)

A square array (SA) contains the integers from 1 to n^2, where n is the order of the array [5,14]. The following are the necessary steps for the construction of OMS from an odd SA, which are given below [10].

i. Consecutive natural numbers 1 to n^2 in n rows and n columns are inserted. Find out the values of the pivot element $P = \frac{1+n^2}{2}$ and the magic sum, $S = \frac{n(1+n^2)}{2}$.
ii. Arrange the $n \times n$ matrix in a BLS so that the column sums are equal.
iii. Select the row associated with P, assign this row as main diagonal elements (keeping the pivot element in the middle cell) in ascending or descending order and arrange the other (column) elements in an orderly manner to get the desired magic square.

As a consequence of the above steps, the Basic Latin Square (considering a 5×5 matrix as an example) is constructed as in Fig. 3.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{22} & a_{23} & a_{24} & a_{25} & a_{21} \\ a_{33} & a_{34} & a_{35} & a_{31} & a_{32} \\ a_{44} & a_{45} & a_{41} & a_{42} & a_{43} \\ a_{55} & a_{51} & a_{52} & a_{53} & a_{54} \end{bmatrix}$$

Fig. 3. Algebraic form of Latin Square

Once the algebraic form of Latin Square is constructed in n^2, then the arrangement of the first magic square is simple. In the above Fig. 3, a_{33} is the pivot element. Representing the pivot element in the middle cell of the 5×5 matrix, we have,

		a_{11}		a_{35}
		a_{22}	a_{34}	
		a_{33}		
	a_{32}	a_{44}		
a_{31}	a_{43}	a_{55}		

Fig. 4. Pivot element in the middle cell

Select the column containing a_{33} and insert it in the middle column of the 5×5 matrix as performed in Fig. 4. Then, we can set up as $[a_{33} \quad a_{44} \quad a_{55} \quad a_{11} \quad a_{22}]$ in the third column i.e. the pivot column.

Again, selecting the column containing a_{34} and arrange it as $[a_{34} \quad a_{45} \quad a_{51} \quad a_{12} \quad a_{23}]$. Similarly, for the columns containing a_{35} , a_{31} and a_{32} we can arrange them as $[a_{35} \quad a_{41} \quad a_{52} \quad a_{13} \quad a_{24}]$, $[a_{31} \quad a_{42} \quad a_{53} \quad a_{14} \quad a_{25}]$ and $[a_{32} \quad a_{43} \quad a_{54} \quad a_{15} \quad a_{21}]$. Finally, we obtain a 5 × 5 magic square as

a_{42}	a_{54}	a_{11}	a_{23}	a_{35}
a_{53}	a_{15}	a_{22}	a_{34}	a_{41}
a_{14}	a_{21}	a_{33}	a_{45}	a_{52}
a_{25}	a_{32}	a_{44}	a_{51}	a_{13}
a_{31}	a_{43}	a_{55}	a_{12}	a_{24}

Fig. 5. Algebraic Magic Square of 5×5

It can be simplified with a numerical example of a 5×5 magic square with the integers 1, 2, 3, ⋯, 25 as follows:

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 7 & 8 & 9 & 10 & 6 \\ 13 & 14 & 15 & 11 & 12 \\ 19 & 20 & 16 & 17 & 18 \\ 25 & 21 & 22 & 23 & 24 \end{bmatrix}$$

Fig. 6. Basic Latin Square of 5×5

Now, $\frac{n^2+1}{2}$ i.e. $\frac{25+1}{2}$ = 13 represents the pivot element keeping row in the diagonal of the 5×5 matrix, we have [15],

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

Fig. 7. Magic Square of 5×5

The constant sum in every row, column and diagonal in the above odd magic square is 65 and it is called the magic constant or magic sum [16]. And this is called the vertical method since the elements in the cells are filled up with vertical wrappings.

3 Our contributions

The first so formed OMS is termed as Base-Magic Square and it is denoted by MS_1 as in Section 2.2. The first column of MS_1 becomes the pivot column of the second magic square which is denoted by MS_2 and so on [11].
The following is the algorithm of odd magic to magic square matrices:

(i) Consecutive natural numbers 1 to n^2 in n rows and n columns are inserted. Find out the values of the pivot element $P = \frac{1+n^2}{2}$ and the magic sum, $S = \frac{n(1+n^2)}{2}$.
(ii) Arrange the $n \times n$ matrix (SA) in BLS to get the column sums equal.
(iii) Select the row associated with P, assign this row as main diagonal elements (keeping the pivot element in the middle cell) in ascending or descending order and arrange the other (column) elements in an orderly manner to get the desired magic square.
(iv) Assign the first magic square so constructed as MS_1 and the elements of MS_1 as $n \times n$ matrix similar to the previous BLS.
(v) Insert the first column of MS_1 as the pivot column in MS_2 and the elements of MS_1 in the cells of $a_{(\frac{n+1}{2})2}, a_{(\frac{n+1}{2})3}, \dots, a_{(\frac{n+1}{2})(\frac{n+1}{2})}$ will replace the diagonal cells of MS_2 as $a_{(\frac{n+1}{2}-1)(\frac{n+1}{2}+1)}, a_{(\frac{n+1}{2}-2)(\frac{n+1}{2}+2)}, \dots, a_{1n}$ and the row elements $a_{(\frac{n+1}{2})(\frac{n+1}{2}+1)}, a_{(\frac{n+1}{2})(\frac{n+1}{2}+2)}, \dots, a_{(\frac{n+1}{2})n}$ on the right of the pivot element of MS_1 will replace the lower diagonal cells of MS_2 as $a_{n1}, a_{(n-1)2}, \dots, a_{(\frac{n+1}{2}+1)(\frac{n+1}{2}-1)}$.
(vi) Repeat step (iii) till all the vacant cells of MS_2 are filled up as performed in the BLS.

(vii) Insert the first column of the previous magic square, the moment at which the vertical method fails to produce magic square, in the horizontal pivot row. Arrange each element accordingly with the horizontal method. And check if the transformed matrix has become a magic square or not.

3.1 Construction of oms by vertical method

As a consequence of the algorithm in Section-3, an Odd SA to an OMS is constructed as in Fig. 7, taking the example of 5×5 matrix

1	2	3	4	5
7	8	9	10	6
13	14	15	11	12
19	20	16	17	18
25	21	22	23	24

Fig. 8. Basic Latin Square of 5×5

The above matrix of 5×5 is a BLS and is transformed to MS_1as in Fig. 9(a).

17	24	1	8	15	65
23	5	7	14	16	65
4	6	13	20	22	65
10	12	19	21	3	65
11	18	25	2	9	65
65	65	65	65	65	65

Fig. 9(a). First generated MS_{51} of 5×5

⇨

21	9	17	5	13	65
2	15	23	6	19	65
8	16	4	12	25	65
14	22	10	18	1	65
20	3	11	24	7	65
65	65	65	65	65	65

Fig. 9(b). Second generated (MS_{52} of 5×5)

Then the MS_{52} of 5×5 can be transformed into the following Fig. 9(c) of the third generated MS_{53} of 5×5.

18	7	21	15	4	**65**
24	13	2	16	10	**65**
5	19	8	22	11	**65**
6	25	14	3	17	**65**
12	1	20	9	23	**65**
65	**65**	**65**	**65**	**65**	**65**

Fig. 9(c). Third generated (MS_{53} of 5×5)

3	23	18	13	8	**65**
9	4	24	19	14	**70**
15	10	5	25	20	**75**
16	11	6	1	21	**55**
22	17	12	7	2	**60**
65	**65**	**65**	**65**	**65**	**15**

Fig. 9(d). Not a MS (5×5)

Therefore, continuing as the above algorithm, we get the following square matrix (SM) which is not a magic (Fig. 9(d)) because sums of 2nd, 3rd, 4th, 5th rows and one diagonal are not equal to magic sum 65. In this way, we get several OMSs.

3.2 Construction of OMS by horizontal method

The horizontal method is applied only when the vertical method is no longer producing magic squares. In this algorithm, the pivot column (the first column) is made the middle row of the next generation matrix and distribution of elements of each column is done row wise in the transformed array with wrapping after the elements of the pivot row is distributed with wrapping as the backward diagonal of the transformed array. The algorithm with horizontal wrapping is applied to the OMSs and non-OMSs generated by the vertical method discussed in the above section taking a 5×5 matrix as an example since a 3×3 matrix produces only one magic square. We take the third generated OMS and fill up the remaining cells as the working principle as follows:

18	7	21	15	4	**65**
24	13	2	16	10	**65**
5	19	8	22	11	**65**
6	25	14	3	17	**65**
12	1	20	9	23	**65**
65	**65**	**65**	**65**	**65**	**65**

Now, all the elements in first column of the OMS Fig. 9(c) [18 24 5 6 12] are inserted as the middle row elements in the following square matrix:

18	24	5	6	12

Then, the middle row elements which include 5 i.e. [5 19 8 22 11]will be distributed diagonally as indicated

14	20	21	2	8
			19	
18	24	5	6	12
	11		23	
22			15	

Furthermore, the other columns which include 6, 12, 18, 24 will be transformed into respective rows. The following figures reflect the activities:

22	3	9	15	16	**65**
10	11	17	23	4	**65**
18	24	5	6	12	**65**
1	7	13	19	25	**65**
14	20	21	2	8	**65**
65	**65**	**65**	**65**	**65**	**65**

Fig. 9(e). Fourth generated (MS_{54} of 5×5)

15	16	22	3	9	**65**
23	4	10	11	17	**65**
6	12	18	24	5	**65**
19	25	1	7	13	**65**
2	8	14	20	21	**65**
65	**65**	**65**	**65**	**65**	**65**

Fig. 9(f). Fifth generated (MS_{55} of 5×5)

3	9	15	16	22	**65**
11	17	23	4	10	**65**
24	5	6	12	18	**65**
7	13	19	25	1	**65**
20	21	2	8	14	**65**
65	**65**	**65**	**65**	**65**	**65**

Fig. 9(g). Sixth generated (MS_{56} of 5×5)

⇨

16	22	3	9	15	**65**
4	10	11	17	23	**65**
12	18	24	5	6	**65**
25	1	7	13	19	**65**
8	14	20	21	2	**65**
65	**65**	**65**	**65**	**65**	**65**

Fig. 9(h). Seventh generated (MS_{57} of 5×5)

9	15	16	22	3	**65**
17	23	4	10	11	**65**
5	6	12	18	24	**65**
13	19	25	1	7	**65**
21	2	8	14	20	**65**
65	**65**	**65**	**65**	**65**	**65**

Fig. 9(i). Eighth generated (MS_{56} of 5×5)

The use of the steps of Section 3 yields that there will be $n-2$ generated OMS from the n-ordered BLS [11]. The remaining magic squares may not have the same pivot element $P = \frac{1+n^2}{2}$because the so-called pivot element has been wrapped around to generate a number of OMSs from the first one. Using the above horizontal method, we conclude that we can generate several magic squares (even for a5× 5 square matrix) using vertical and horizontal algorithms.

By the above horizontal method, we conclude that there are5 several magic squares in 5× 5 square matrix.

4 Comparison

In Section 5, we have constructed a program by which one is free to generate several OMSs. One interesting question is how many OMSs can be generated. In Section 2, only 1 (one) Magic Squares is possible from a 3×3 matrix and similarly in Section 3, we could generate only 3 OMSs from a 5×5 array. Let us see the expectation of a number of OMS from 7×7 OMS.

Fig. 10(b) is the first generated Odd Magic Square of 7×7 order. Then assuming Fig. 10(b) as the

01	02	03	04	05	06	07
9	10	11	12	13	14	08
17	18	19	20	21	15	16
25	26	27	28	22	23	24
33	34	35	29	30	31	32
41	42	36	37	38	39	40
49	43	44	45	46	47	48

⇨

30	39	48	01	10	19	28
38	47	07	09	18	27	29
46	06	08	17	26	35	37
05	14	16	25	34	36	45
13	15	24	33	42	44	04
21	23	32	41	43	03	12
22	31	40	49	02	11	20

Fig. 10(a). Basic Latin Square of 7×7

Fig. 10(b). First generated (MS71 of 7×7)

Basic Latin Square matrix, we proceed as follows:

42	03	20	30	47	08	25
43	11	28	38	06	16	33
02	19	29	46	14	24	41
10	27	37	05	15	32	49
18	35	45	13	23	40	01
26	36	04	21	31	48	09
34	44	12	22	39	07	17

⇨

23	48	17	42	11	29	05
31	07	25	43	19	37	13
39	08	33	02	27	45	21
47	16	41	10	35	04	22
06	24	49	18	36	12	30
14	32	01	26	44	20	38
15	40	09	34	03	28	46

Fig. 10(c). Second generated (MS72 of 7×7)

Fig. 10(d). Third generated (MS73 of 7×7)

Still, we are able to score 3 magic squares. From the third generated 7×7 magic square we can generate two more magic squares:

36	20	46	23	07	33	10
44	28	05	31	08	41	18
03	29	13	39	16	49	26
11	37	21	47	24	01	34
19	45	22	06	32	09	42
27	04	30	14	40	17	43
35	12	38	15	48	25	02

⇨

32	17	02	36	28	13	47
40	25	10	44	29	21	06
48	33	18	03	37	22	14
07	41	26	11	45	30	15
08	49	34	19	04	38	23
16	01	42	27	12	46	31
24	09	43	35	20	05	39

Fig. 10(e). Fourth generated (MS74 of 7×7)

Fig. 10(f). Fifth generated (MS75 of 7×7)

Further, if we proceed with the same programme, it is not possible to get another magic square different from what we have already generated. Hence, we could generate only 5 magic squares from the 7×7 odd magic square with the vertical method. Here, MS_{75} represents the fifth odd magic square generated from the 7×7 odd magic square matrix. Continuing the process of horizontal method from Fig. 10(f), we could generate another 7 OMS.

5 Computation of OMSs

Following the algorithm that we have described in Section 3, a programme for the computation of Magic Square Matrices can be writtenin FORTRAN programming language and running it we have tested the results of our algorithms and we found an interesting relationship between the orders of the matrices with the number of magic squares, and our finding are summarised as follows from the examples we have discussed in the previous sections:

(i) As an example, the total number of all possible OMS for 7×7 NMS is found to be equal to 12 (that is 5 + 7 = = 2 × (7 - 1)) by running our FORTRAN code.
(ii) This means that from an n-ordered NMS, we can generate (n-2) OMSs from the vertical method and n OMSs from the horizontal method, i.e. $n - 2 + n = 2n - 2 = 2(n - 1)$. And generalizing this result, we can say that 2(n-1) odd magic squares can be constructed from an n-ordered NMS.
(iii) Several magic squares generated from an*n*-ordered NMS will be helpful to find the points of inflection on elliptic curves so that the decryption technique of a message may become a complex one [17]. This programme has different behaviour in 3×3 and other odd NMSs the orders of which are multiples of 3, such as (9×9), (15×15), (21×21), etc. which are deviated from the normal pattern [18-20].

6 Conclusions

An improved technique for the generation of OMSs from odd NMSs has been proposed. With the improved algorithm we have developed, it is observed that the new technique could generate 2(*n*-1) number of OMSs from *n*-ordered NMSs. In an earlier work of ours, it was concluded that with the vertical method, only (*n*-2) OMSs from each n-ordered NMSs could be generated. However, there is still an exception for 3*n*-ordered (*n* being a natural number here) NMSs as the total number of OMSs cannot be expressed as 2(*n*-1). The enhanced algorithm can be employed in encryption of big data that cannot be decrypted easily.

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Competing Interests

Authors have declared that no competing interests exist.

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