economies

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A New Approach to Measuring Tax Effort

Basil Dalamagas *, Panagiotis Palaios and Stefanos Tantos

Department of Economics, National and Kapodistrian University of Athens, 1, Sofokleous & Aristidou str., 105 59 Athens, Greece
* Correspondence: dalamaga@econ.uoa.gr; Tel.: +30-210-368-9436

Received: 18 December 2018; Accepted: 25 June 2019; Published: 5 August 2019

Abstract: This paper attempts to extend the theoretical and empirical methodology employed in previous literature, by proposing a utility maximization process to estimate the optimal tax revenue from a sample of 30 countries. It is shown that an optimal tax system is defined solely by two crucial determining factors: The productive capacity of the country (GDP) and consumers' preferences (consumption spending). All the other variables can be disregarded, as macroeconomic determinants (GDP, consumption) tend to capture the impact of all the remaining factors on tax revenue. It is also shown that our utility maximization method generates tax-effort indices which do not differ significantly from those of IMF and World Bank studies. The actual tax burden for most of the sample countries is shown to be below its optimal level. As expected, the tax-effort performance of each of the sample countries appears to be affected by the variety of approaches employed throughout the text.

Keywords: tax effort; tax capacity; utility maximization; optimal tax revenue

JEL Classification: H30; H61; E62

1. Introduction

The main body of literature (Bahl 1971; Chelliah et al. 1975; Crivelli and Gupta 2014; Cyan et al. 2013; Garg et al. 2017; etc.) on tax effort focuses on identifying the forces—in terms of administrative capacity and political, social or economic principles—which determine the capability of policy makers to impose taxes.

A large number of determinants of the tax effort are available for empirical testing, across a broad sample of developing and developed countries and over an adequate number of time periods (Gupta 2007; Kim 2007; Le et al. 2012; Stotsky and WoldeMariam 1997; Langford and Ohlenburg 2016; Leuthold 1991; Fenochietto and Pessino 2013; etc.). In estimating the tax effort regression equation, two methods are used to find an observable proxy for the unobserved dependent tax-effort variable, namely, the average tax ratio method and the potential tax revenue method. According to the first method, the ratio of tax revenue to GDP (tax share) is taken to be a good proxy for tax effort. In the potential tax revenue method, a multiple regression analysis is used, where the tax share is defined as the ratio of the actual tax revenue to tax capacity, proxied by GDP.

In both cases, the tax effort is defined as the ratio of the actual value of the dependent variable to the estimated (predicted) value (Kim 2007). Since, however, the average tax ratio and the potential tax revenue methods provide biased estimators for tax effort, Kim (2007) applies the Kalman filter estimation technique to overcome this problem.

A number of empirical studies have been carried out over the last fifty years to evaluate the effect of a variety of determining factors (agriculture, per capita income, mining, manufacturing, foreign trade, political instability, corruption, etc.) on tax revenue. See, for example, (Lotz and Mors 1967;

Chelliah et al. 1975; Tait et al. 1979; Oates 1988; Dollery and Worthington 1996; Cukierman et al. 1992; Edwards and Tabellini 1991; Alesina and Drazen 1991; Przeworski and Limongi 1993).

Another set of empirical studies focuses on the ratio of actual tax revenue to GDP, that is interpreted as a measure of tax effort and used as the basis for cross country tax comparison. The above ratio is a reasonable fiscal indicator to establish trends or to compare tax revenue performance across countries with similar economic structures and/or levels of income. However, when comparisons of effectiveness in revenue mobilization are to be made across countries with different income levels, the ratio of actual tax revenue to GDP could lead to misleading results due to differing economic structures, institutional arrangements and demographic trends. In this case, the ratio of actual tax revenue of a country to its estimated taxable capacity is considered to be a more convincing tax effort index, capable of capturing each country's specific fiscal, institutional and demographic characteristics.

In the area of taxable capacity and the corresponding definition of tax effort, a number of studies have been conducted, mainly by World Bank and International Monetary Fund (IMF). They calculate the predicted tax to GDP ratio using the estimated coefficients of a regression specification that takes into account country specific characteristics. A "high tax effort" in excess of one implies that the country well utilizes its tax base to collect tax revenue, whereas a "low tax effort" taking a value lower than one indicates a potentially strong role of the tax collection agency in mobilizing domestic sources of funds. The same technique applies to estimating the fiscal revenue capacity (tax plus non-tax revenue) and the corresponding fiscal revenue effort. The results of the most important of these studies which fall into what is known as the standard regression approach, as well as the results of three alternative approaches (stochastic frontier approach, welfare maximization approach and budget balance approach), are described, analyzed and criticized in Appendix A.

The rationale behind our work stands in marked contrast to that of all previous research on measuring tax effort. In particular, a novel idea is employed in the present paper beyond the conventional concept of predicted tax revenue stemming from regression techniques, as they have been derived in previous literature. The main innovation of this paper is that it makes an attempt, through a utility maximization process, to overcome the problems which arise in the empirical estimation of taxable capacity, namely the problems associated with the choice of appropriate explanatory variables and the validity of the datasets employed.

To this end, the present paper employs in Section 3 a cross-country study to estimate the optimal tax revenue from a sample of 30 countries during the period 1996–2015 and the two subperiods of 1995–2009 (before the eruption of the world financial-economic-debt crisis) and 2010–2015 (after the crisis). The optimal tax revenue is derived from a utility maximization process, where the government chooses the proper set of public goods to maximize social welfare, subject to the constraint that the direct-indirect tax system will yield sufficient revenue to finance the required level of government spending. The results obtained from adopting our approach show that the potential tax capacity of private agents is better captured by the difference between GDP and consumption.

An executive summary that facilitates a quick and comprehensive understanding of the key points addressed by the present study is given below:

(1) In the context of Arrow-Debreu economy, with fixed labor supply and no savings, so that disposable income (Y) is equal to private consumption (C), a utility function with two arguments (income and government spending) is maximized with respect to direct and indirect tax rates. The first-order conditions are then manipulated to provide the optimal tax revenue as the difference between income and consumption ($T^* = Y - C$).

(2) The most commonly adopted definition of tax effort is used, i.e., the ratio of actual to optimal tax revenue (T/T^*), to determine the state of overtaxing the economy ($T/T^* > 1$) or undertaxing it ($T/T^* < 1$).

(3) The methodology employed in the present study is in sharp contrast to the claim of all previous researchers that the optimal tax revenue is derived, directly or indirectly, from the fitted (predicted) values estimated from running a regression of (actual) tax revenue on a large number of economic

variables (GDP, imports, exports, mining, agriculture, state of development, population, etc.) and unobserved or non-economic factors (political, institutional, demographic, geographical, ethical and legal indices, cultural and so on).

(4) The drawback of the standard regression analysis employed by previous researchers is that there is no adequate a priori justification for an ad hoc use of variables selected as measures of taxable capacity, whereas data, especially on non-economic factors, are not reliable. Thus, what previous empirical works reveal is that, until now, no serious attempts have been made at giving a theoretical background to tax effort econometric estimates.

(5) In contrasting the theoretical model employed in the current paper against alternative models, it is not difficult to clearly identify the value added of the present study: The optimal level of tax revenue is derived from a utility maximization process and is shown to be equal to the difference between income and consumption, regardless of the prevailing economic, institutional, political and other conditions in each country. All these economic and non-economic factors of previous studies are assumed to have been incorporated in the private and public agents' priorities, regarding work effort and consumers' preferences.

(6) Our finding that optimal tax revenue is equal to Y-C, i.e., it is equal to private savings, is in conformity with the no-savings assumption of the Arrow-Debreu economy. In a closed economy with no savings, the only way for the government to raise revenue up to the optimum is through the taxation of savings.

(7) Our estimates of the tax effort show that, the majority of the countries examined (26 out of 30) have a tax-effort index lower than one (average 0.81), pointing to an actual tax revenue lagging behind the corresponding optimal one.

(8) In the rest of our paper, one can compare our estimates on tax effort to those of previous studies. The comparison reveals that there are no substantial differences between the two categories of estimates, at least at the average level.

The rest of the paper is organized as follows. Section 2 contains a discussion of the estimation results for each of the above four approaches. These results are then compared with the results of applying the standard regression approach to the set of the 30 countries contained in our model, to trace out possible discrepancies. In Section 3, we propose a new measure of tax potential, in the context of a utility maximization model, and draw a comparison between the tax effort index that is estimated in the present paper and the tax effort indices calculated by using the four approaches employed in existing literature. Section 4 concludes the discussion laying out directions for further work.

2. Experimenting with Prevailing Approaches to Tax Effort

Before proceeding with the description of our model, it would be advisable to agree to a necessary condition as a prerequisite of dealing with the soundness of the proposed new approach to measuring tax effort. Otherwise, it would be quite difficult for the reader to understand the degree of reliability and the implications of our model, as well as to evaluate the differences between our methodology and the procedures followed by previous researchers. This condition has to do with summarizing the main characteristics and findings of the research methods employed by previous contributions to the subject matter and then to contrast them with the estimation results of our model.

Thus, in the first place, we replicate the standard regression analysis to determine the tax effort for a set of 30 countries. This set forms part of the sample countries employed by the most important contributions to tax effort. The regression equation is estimated using a panel of data covering the time period 1996–2015. The goal is to measure the tax effort (ratio of actual tax revenue to predicted tax collections) for the set of 30 countries and to compare it with the corresponding tax effort determined by previous studies.

As already stressed, the determination of a country's tax effort by regressing the actual tax effort on a variety of explanatory variables (and instruments) is not considered to be an essential ingredient of any lasting solution leading to a viable (direct and indirect) tax system. However, it is of crucial importance for the purpose of the present study to be able to compare the findings of previous researchers (including our findings in this section) with the results to be obtained under the new approach that will be presented in the next section.

The econometric technique that is applied in this section, when evaluating the predicted values of tax effort, is the standard regression analysis. All the explanatory variables used in previous literature are tested, and the statistically significant ones are chosen to estimate the tax effort index. A subset of 30 countries common in all previous studies is selected, with the only difference being in the time-period covered. A detailed description of data and sources employed in this section is given in Appendix B. We experimented with a large number of combinations containing explanatory variables, drawn from previous studies, on the basis of the following equation,

$$T_{it} = a + \beta X_{it} + e \quad (1)$$

where T is tax revenue as a percentage of GDP, with i = 1, … , N and t = 1, … L being country and time indicators, respectively, a is the positive constant and e the error term, the properties of which are described below. Equation (1) is regressed on X which includes per capita GDP, age dependency, imports, the share of agriculture in GDP, the level of corruption and political stability. Other control (structural, institutional and policy) variables examined (but not selected) include inflation, external indebtedness, foreign aid in relation to GDP, fiscal deficits, growth rate etc.

The sample data are represented by 30 cross-country units over 20 periods of time. Hence, there are 600 observations. The data with the corresponding sources are given in Appendix B.

Following the procedural steps adopted by Leuthold (1991), the model is first estimated using ordinary least squares (OLS) regression on the pooled data. The dependent variable (tax revenue), imports and agriculture are ratios (percentage of GDP), while age dependency is also a ratio (percentage of working age population). Since both the numerator and denominator of the ratios are measured in the country's own currency, no deflation or conversion into a common currency is necessary. Per capita income is calculated by first deflating GDP to constant 2010 prices, using each country's own GDP deflator. Then, it is converted into US$, using the current year's exchange rate and put on a per capita basis by dividing by population. Corruption and political stability are indices and, hence, there is no need for any kind of transformation.

The regression model, according to Leuthold (1991), is estimated mainly as a benchmark model. The regression model does not take into account possible correlation in the error term across countries and over time. This is why an autoregressive model will be employed.

The autoregressive model allows for the fact that the model may be first-order autoregressive with contemporaneous correlation between cross sections by assuming that the errors are random,

$$e_{ij}, \; i = 1 \ldots 30, \text{ and } j = 1 \ldots 20,$$

and have the following structure:

$$E(e^2{}_{ij}) = s_{ii} \text{ (heteroscedasticity)},$$

$$E(e_{ij}e_{kj}) = s_{ik} \text{ (contemporaneously correlated)},$$

$$e_{ij} = r_i e_{i,j-1+} u_{ij} \text{ (autoregression)}$$

Heteroscedasticity may arise if there is a higher variance in the estimated residuals for the larger countries in terms of population or income. Contemporaneous correlation arises if the countries are from the same geographic region, and autocorrelation arises if the error this period in country i depends on the error last period in country i. The model is estimated using generalized least squares on the transformed data. Data transformation takes place according to the model described by Kmenta (1986, pp. 622–25). The results are presented in Table 1.

As shown in Table 1, the regression model performs poorly. Only imports and GDP per capita have the expected sign, whereas the GDP per capita is statistically insignificant. Moreover, the adjusted R^2 is low and the Durbin-Watson statistic (DW = 0.05) indicates rejection of the null hypothesis of no autoregression at the 1% confidence level. The above drawbacks of the regression model are sufficient to justify our decision to focus on evaluating the coefficient estimates derived from the autoregressive model (specifically, the estimates presented in column 2, Table 1).

The tax capacity of countries is defined as the fitted values calculated using the estimated coefficients reported in Table 1, column 2. The sample includes 30 countries, resulting in a panel data set of 420 observations over the 14-year subperiod from 1996 to 2009. Our dataset covers the period 1996–2015. However, for comparison reasons, we have employed the subperiod 1996–2009, which corresponds to the period selected by the IMF, World Bank and stochastic frontier models.

A first finding is that the coefficients of the autoregressive model are statistically significant and have the expected signs. Per capita income, openness and political stability have a strong positive relationship with revenue performance, whereas the levels of age dependency, corruption and agricultural share have a strong negative effect on the revenue to GDP ratio. Finally, the adjusted R-squared is as high as 0.83, indicating a very good fit of the empirical specification.

Table 1. Model estimation results using pooled data, (1996–2009).

Dependent Variable: Tax Revenue (% of GDP)	**Regression Model (1)**	**Autoregressive Model (2)**
GDP per capita (constant 2010 US$)	0.000009 (0.29)	0.00003 *** (3.40)
Age dependency ratio (% of working-age population)	0.36 *** (3.67)	−0.17 *** (−2.66)
Imports of goods and services (% of GDP)	0.08 *** (4.52)	0.15 *** (4.08)
Agriculture, value added (% of GDP)	0.84 *** (2.76)	−0.16 ** (−2.48)
Corruption	2.72 *** (2.92)	−2.34 *** (−3.08)
Political Stability and Absence of Violence/Terrorism	−3.27 *** (−2.39)	0.55 ** (2.27)
Constant	−5.21 (−0.98)	26.12 *** (11.08)
Adjusted R-squared	0.18	0.83
Number of Observations	420	420

** Indicates significant at 5 percent level; *** Indicates significant at 1 percent level. *t*-statistics are in parentheses.

To check the robustness of the empirical results, we have run alternative specifications, by adding shadow economy and total consumption (as a percentage of GDP), once at a time, to the explanatory variables.

As the size of shadow economy increases, it gets harder to track personal income, profits and sales, and thus, the tax collection mechanism becomes less efficient. In contrast, higher consumption is expected to have a positive effect on tax collection, mainly through higher indirect taxes. Even though the shadow economy was found to be a statistically significant and negative determinant of taxes and consumption proved to be a positive and significant factor, our empirical results are robust to the inclusion of these additional variables (analytical results are available by the authors on request).

Using the coefficient estimates of Table 1 (column 2), we calculate the tax effort indices for the sample of 30 countries. The results are presented in Table 2 (column 1). According to the ranking of countries on the basis of their tax effort, Finland, Cyprus and Sweden (column 1) are shown to have the highest tax effort. Half of the sample countries (15) are shown to have a tax effort score above unity, and 15 countries have a tax-effort score lower than 1. Remember that countries with a tax effort index less than unity tend to exploit their estimated tax potential less than the average, thus making limited use of their tax bases to increase revenue, and vice versa. Our empirical analysis also shows that the stage of development plays no role in the determination of tax effort. Some developed countries (Belgium, Denmark, Australia, Finland, Sweden, UK etc.) are above their tax capacity, having a high tax effort, while other developed countries (Germany, Switzerland, USA, Japan etc.) have a low tax effort.

Table 2. Tax effort indices in the context of the approaches of standard regression, stochastic frontier and balanced budget.

Standard Regression Indices					
	Own Estimated Index 1996–2009 (1)	**WB Index 1994–2009 (2)**	**IMF Index 1991–2012 (3)**	**Stochastic Frontier Index (4)**	**Balanced Budget Index (5)**
Belgium	1.15	1.24	0.94	0.91	0.98
Bulgaria	0.65	0.98	0.69	0.93	1.01
Czech	0.58	0.98	0.78		
Denmark	1.26	0.99	0.96	1.07	1.04
Germany	0.68	0.87	0.83	0.89	0.96
Ireland	1.14	0.97	0.67	0.71	0.91
Greece	1.24	1.14	0.81		
Spain	0.78	0.90	0.81	0.80	0.97
France	1.26	1.29	0.97	0.95	0.91
Croatia	0.82	1.18	0.81	1.06	
Italy	1.18	1.25	0.99	1.00	0.90
Cyprus	1.32	1.40	0.69	0.96	0.67
Latvia	0.78	0.80	0.64	0.96	0.91
Luxemburg	1.23	0.94	0.85	0.64	1.01
Hungary	0.76	1.12	0.86	0.77	0.84
The Netherlands	1.12	1.10	0.86	0.84	0.98
Australia	1.25	1.09	0.97	0.67	1.03
Polland	0.68	1.02	0.78	0.86	0.86
Portugal	0.88	1.03	0.74	0.87	0.81
Romania	0.65	0.84	0.67	0.79	0.99
Slovenia	0.77	1.13	0.77	0.76	0.95
Slovakia	0.44	0.89	0.71	0.91	0.86
Finland	1.32	1.05	0.96	0.85	1.06
Sweden	1.43	0.98	0.98	0.85	1.01
United Kingdom	1.23	1.10	0.85	0.82	0.92
Iceland	1.28	0.90	0.76	1.06	0.99
Norway	1.25	1.13	0.92	0.85	1.09
Switzerland	0.83	0.56	0.69	0.64	0.94
USA	0.85	0.77	0.71	0.89	0.89
Japan	0.54	0.47	0.67	0.74	0.80
Average tax effort	**0.98**	**1.00**	**0.81**	**0.86**	**0.94**

Column 1: Tax effort = The ratio of actual tax revenue to the fitted values from running the autoregressive model (see Table 1, column 2); Column 2: Le et al. (2012) Tax Capacity and Tax Effort: Extended Cross-Country Analysis from 1994 to 2009. World Bank Policy Research Working Paper No: 6252; Column 3: Fenochietto and Pessino (2013) Understanding Countries' Tax Effort. IMF Working Paper WP/13/244; Columns 4 and 5: Cyan et al. (2013) Measuring Tax Effort: Does the estimation approach matter and should effort be linked to expenditure goals? International Center for Public Policy Working Paper 13-08. Note: There are no estimates in the literature for the welfare maximization approach.

In general, the "own estimates", in Table 2 appear to be quite different, typically falling outside the range of other estimates (sometimes above, sometimes below) and the implied ranking of countries looks like it would be quite different. It is evident that mixed correlation results arise when "own estimates" of tax effort are compared to those of the stochastic and the balanced budget approaches as shown in Table 3. Note, however, that the Spearman and Pearson correlations between "own estimates" and those of the IMF and World Bank are high, pointing to observations having a similar rank, as will be discussed below.

To look at the problem from another aspect, the estimates of the other models seem more consistent with each other than they are with the "own estimates", suggesting that the latter offer no improvement. It should be stressed, however, that the aim of the present study is not to produce results which align the own estimates with those of previous studies. It is rather to introduce a completely novel approach of calculating the tax effort and, as such, it should be judged on its own merits and not on the basis of estimation (dis)similarities between the two cases.

The next step in our analysis is to compare the results of our estimates on tax effort (Table 2, column 1) with those obtained by World Bank (column 2), IMF (column 3) and International Center for Public Policy (columns 4 and 5). All these studies relate to the same sample of countries, but they cover a slightly diverse time period. Comparisons will be carried out by employing the correlation coefficients of Pearson and Spearman: Pearson's correlation analysis (centered and uncentered) and Spearman's rank-order covariance analysis[1]. The results are presented in Table 3.

Table 3. Comparing the results of various studies on tax effort.

Present Study, Augmented Model			
	Pearson Centered (1)	**Pearson Uncentered (2)**	**Spearman Rank Order (3)**
IMF (standard regression)	0.53 (0.0071)	0.94 (0.00)	0.51 (0.004)
World Bank (standard regression)	0.66 (0.0002)	0.95 (0.00)	0.63 (0.0006)
Stochastic frontier	0.31 (0.17)	0.96 (0.00)	0.24 (0.32)
Balanced Budget	0.22 (0.33)	0.95 (0.00)	0.41 (0.06)

Two points must be noted in the interpretation of these results:

- The Spearman and Pearson correlations between our estimated series of tax effort and each of the IMF and World Bank corresponding series are high, pointing to observations having a similar rank (or similar raw numbers) between each of the pairs of tax effort. Such a finding leads to the tentative conclusion that the standard regression analysis employed in calculating the tax effort may possibly give similar results, regardless of the explanatory variables used by each researcher.
- Mixed correlation results arise when our estimates of tax effort are compared to the corresponding estimates of both the stochastic frontier and the balanced budget approaches: The statistical dependence between the ranking (or raw numbers) of each of the four pairs of tax effort is significant in three of the cases considered and insignificant in the remaining three cases.

It becomes evident that there are statistically significant differences in the values of tax effort, depending on the methodology used (regression analysis or stochastic frontier approach) to calculate this index. The most likely explanation is that the stochastic frontier approach combines elements

[1] Spearman's rank correlation coefficient is a widely used nonparametric measure of rank correlation (statistical dependence between the ranking of two variables) and describes the relationship between two variables by employing a monotonic function (whether linear or not). It is equal to the Pearson correlation, which however assesses only linear relationships. Specifically, the Pearson product-moment correlation coefficient measures the linear relationship between the raw numbers rather than between their ranks. A perfect Spearman correlation of +1 or −1 occurs when each of the variables is a perfect monotonic function of the other.

from both the standard regression analysis and an incomplete utility maximization process. However, regardless of the differences in the estimated values of tax effort, the major problem from all methods employed in previous tax-effort literature is that they do not lead to a Pareto optimal outcome.

If Pareto efficiency should come to the fore, the natural question would arise as to whether a distortionary tax system could lead to a Pareto optimal tax revenue. Normally, an optimal outcome would be expected to occur if the government could maximize social welfare from a combination of direct (equitable but inefficient) and indirect (inequitable and inefficient) taxes, subject to the constraint that sufficient revenue will be achieved to finance the provision of public goods.

This observation motivates the study of both finding a welfare maximizing level of tax revenue and comparing the results to the values of tax effort presented in Table 2. Allowing for a properly designed tax system, it would sound interesting to extend the Arrow-Debreu economy and to modify slightly the Samuelson rule. The latter assumes that the cost of supplying public goods should be funded by a neutral lump-sum tax that does not distort individual preferences. However, optimal lump-sum taxes can rarely be employed in practice because they are inequitable, even though they are efficient and non-distortionary. These properties of lump sum taxes stimulate interest in looking for alternative forms of finance, with a system of distortion-minimizing progressive (equitable but inefficient) direct taxes and regressive (inequitable and inefficient) indirect taxes. Such a system would both maximize social welfare and help government to determine an optimal level of tax revenue, with decentralization of the provision of private goods.

3. The Model: Results and Discussion

The analysis of Section 3 is based on the concept of a competitive general equilibrium economy, as described in the seminal contribution of Arrow and Debreu (1954). Lacking capital, our model is static with fixed labor supply and no savings, assuming that GDP is exhausted by private consumption and government spending. The period preference function is log-additive, in the spirit of Samuelson's utility functions for public goods, as described by G. Myles (1997, chp. 9). The relevant literature postulates that tax revenue is used by the government to purchase an amount of labor input of equal value, in order to produce non-tradable goods, such as defense services.

In what follows, we employ two discrete models, one with proportional taxes (Section 3.2) and the other with a progressive, income tax and a regressive consumption tax (Section 3.3). Both models lead to the same conclusion: an optimal tax revenue is one that exhausts the difference between income and private consumption, $T = Y - C = G$, whereas the tax effort is defined as the ratio of actual tax revenue to the optimal one, $\frac{T}{Y-C}$ averaged over selected time spans, thus capturing the steady state implicit in our model.

Two notes should be made with respect to our results:

(1) A balanced government budget is postulated throughout our analysis, in conformity with the simple version of the Arrow-Debreu model. In addition, the absence of saving and investment also implies a constraint of the form $Y = C + G$. Thus, the possibility of starting from an initial no-equilibrium position, $G(= T) \neq Y - C$, or the possibility of issuing public debt are not considered in the present text. Note that the omission of the government debt would not lead to any qualitatively different definition of the tax effort, because the effect of deficit, in the form of a constant variable, $(G - T)$, would be eliminated in the maximization process.

(2) The underlying assumption is that capital, investment and a motive for saving are missing from our model. To overcome this problem, we take as given that the government engages in a kind of forced saving, collecting an income tax (T_y) from the household's income and levying a commodity tax, T_p, on net (before tax) consumption, thus taking a portion of the taxpayer's income/consumption, that he would otherwise save. In this case $Y - T_y = C + T_p$ is the correct national income identity, since $T = T_y + T_p = G(= Y - C)$.

The assumption of forced savings allows us to replace the initial GDP definition of households' income with the concept of disposable income, described as the difference between GDP and direct taxes. It also allows us to introduce the concept of real (after direct and indirect taxes) disposable income, defined as the ratio of disposable income to the adjusted inflation rate. Starting from a no-indirect tax position and zero inflation rate, the adjusted inflation rate is assumed to be augmented by the commodity tax rate. This, in turn, leads to the notion of adjusted consumption, described as the real purchasing power that remains after the increase in the price level due to the introduction of indirect taxes.

Last but not least, the equivalence between disposable income and gross (including indirect taxes) consumption paves the way for treating interchangeably these two variables in Section 3.2—where disposable income is the first argument of the utility function in Equations (10) and (12)—and Section 3.3—where gross consumption is the first argument of the same function in Equation (22).

3.1. The General Framework

To provide a reasonably simple derivation of an optimal tax revenue, it will be assumed that we have an Arrow-Debreu economy with no saving and fixed labor supply, consisting of H households with identical preferences. A utility function that represents the state's preferences is written

$$U = U(Y, G) = \ln Y + \ln G \tag{2}$$

where Y is GDP and G is the supply of public goods and services. It is further assumed that the allocation of resources to private and public goods, i.e., the combination of these goods that the economy produces, depends on the fiscal choice of policy makers regarding the amount of productive resources which will be moved via taxation into the provision of public services, given the production possibilities of the economy. This assumption is necessary because market clearing implies that the revenue constraint and the production constraint may be used interchangeably.

To characterize the set of first-best, or Pareto efficient, allocation, the government chooses the vectors of income (consumption) and public goods to maximize social welfare, constrained by the requirement that the tax system should yield sufficient revenue to finance the provision of public goods. The tax system, T, consists of a progressive direct tax (personal income tax), T_y, and a proportional indirect (commodity) tax, T_ρ. The representation of the government budget constraint is written

$$G = T = T_y + T_\rho \tag{3}$$

Note that in (3), the public borrowing capacity has been omitted. This is in conformity with the crucial assumption of no-saving and no government borrowing assumptions adopted by the competitive general equilibrium theory of Arrow-Debreu. Note also that no allowance for the preference for public goods has been made. This is because, in the context of a general equilibrium analysis, government intervention by assumption does not distort consumers' preferences for traded goods in private markets. As discussed above, fiscal authorities are taken to use tax revenue in order to purchase labor units of equal value and to employ them in the provision of non-traded public goods, e.g., for defense purposes.

A progressive personal income tax is defined as a set of successively increasing marginal tax rates, each of which applies to a particular income bracket, y, of the household, with the sum of these brackets corresponding to the income-tax base,

$$T_y = \sum_{j=1}^{m} t_{y,j} y_j \tag{4}$$

where $t_{y,j}$ stands for the marginal tax rate of the j income bracket, $j = 1, \ldots, m$. The average tax rate is written

$$\bar{t}_y = \frac{\sum_{j=1}^{m} t_{y,j} y_j}{\sum_{j=1}^{m} y_j} \tag{5}$$

In general, the main features of the income tax[2] may be better captured by a number of appropriate functional forms, such as $T_y = \beta Y^2$, where $0 < \beta < 1$, because the marginal tax rate, $t_{y,j} = \frac{dT_y}{dY} = 2\beta Y$, is greater than the average tax rate, $\bar{t}_y = \frac{T_y}{Y} = \beta Y$.

Moreover, both the average and marginal tax rates increase with income and income-tax elasticity, $e = \frac{dT_y}{dY}\frac{Y}{T_y} = \frac{t_{y,j}}{\bar{t}_y} = 2$, is greater than one. Therefore, the progressive income-tax function to be used in the present text takes the form:

$$T_y = \bar{t}_y Y = \beta Y^2, \text{ with } \beta = \frac{\bar{t}_y}{Y} \text{ (or } \bar{t}_y = \beta Y) \tag{6}$$

The second tax category to finance the provision of public goods is commodity taxation. One of the properties of indirect tax rates is that they are proportional with respect to the tax base (consumption spending, C),

$$T_\rho = t_\rho C \tag{7a}$$

where t_ρ is the proportional commodity tax rate and, simultaneously, they are regressive with respect to income,

$$T_\rho = t_r Y \tag{7b}$$

where t_r is the regressive commodity tax rate.

An indirect tax is regressive with respect to income because the ratio of consumer's expenditure to income [average (or marginal) propensity to consume, c] falls as income rises, i.e.,

$$c = \left(\frac{C}{Y}\right) = \frac{\gamma}{Y^2}, \text{ or } t_\rho c = \frac{t_\rho C}{Y} = \frac{t_\rho \gamma}{Y^2} \tag{8}$$

where γ is a constant, $\gamma > 1$.

Combining (7a), (7b) and (8) gives:

$$T_\rho = t_\rho C = t_r Y, \text{ or } \frac{t_r}{t_\rho} = \frac{C}{Y} = \frac{\gamma}{Y^2}, \tag{9a}$$

and

$$t_r = t_\rho \frac{\gamma}{Y^2}, \text{ or } t_\rho = t_r \frac{Y^2}{\gamma}, \text{ or } \gamma = \frac{t_r}{t_\rho} Y^2 = \frac{C}{Y} Y^2 = CY \tag{9b}$$

In (9b), the regressive indirect tax rate is shown to increase as income decreases.

The tax system, as defined above, is shown to consist of two classes of distortionary taxes. The personal income tax is equitable (households with lower income pay a lower percentage of their

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[2] The main features of the personal income tax are the following:

- Both the marginal and the average tax rates increase as income increases,
- the marginal tax rate is higher than the average tax rate, and
- the income elasticity of the tax is greater than one.

income in tax than do those with higher income) but inefficient (it provides a disincentive to work harder). On the other hand, commodity taxation is inequitable (the marginal propensity to consume and, hence, the tax to income ratio rise as income falls) and inefficient (it discourages consumption). The question then arises how can these taxes fit into the framework of a Pareto optimizing procedure that requires tax neutrality.

In general, when the public good has to be financed by distortionary taxation, there is a divergence of the marginal rate of transformation (MRT) from the sum of the marginal rates of substitution (ΣMRS). This divergence leads to a benefit measure that is less than the ΣMRS. The question of whether more or less public good should be provided, when its provision is financed by distortionary taxation, has not been directly answered (see Myles 1997, chp. 9). Since no complete answer has been given, an alternative perspective upon this issue will be presented in the present framework, based on three assumptions.

The first assumption is that the inequitable nature of commodity taxation is exactly balanced with the equitable properties of the personal income tax. This may occur, for example, with a properly designed structure of the personal income tax (marginal income tax rates and income brackets) that would result in a lower tax burden for the low income groups and would offset the heavier indirect tax obligations on them. Restricting direct and indirect taxes to have offsetting distortionary characteristics from an (in)equity point of view may not allow the first-best to be achieved, but it may lead to a second-best outcome.

The second assumption is that, with the fixed labor supply postulate adopted in our analysis, the personal income tax does not cause any substitution effect between leisure and hours worked, so work incentives are not affected by direct taxes.

The third assumption is that the commodity tax system is designed along the lines suggested by the Ramsey Rule: The optimal tax system should be such that the compensated demand for each good is reduced in the same proportion relative to the pre-tax position. In this case, the distortions in terms of quantities can be minimized, given that, in a Ramsey's environment, it is the level of consumption that actually determines welfare. What happens to prices is unimportant. In this context, commodity taxation does not disturb individual preferences between consumption goods and/or between consumption and savings.

The above assumptions may be seen as a response to the argument of non-optimality of the market equilibrium, in the absence of optimal lump-sum taxes. In formalizing this response, we argue that a natural policy to adopt in order to achieve optimality would be the employment of an appropriate set of (direct-indirect) taxes to correct for the distortions. Such a tax structure, in the spirit of Pigouvian taxes, is postulated to hold in the design of the models in Sections 3.2 and 3.3.

Having defined the theoretical framework and the set of tax instruments to be used in our analysis, we can move on to the formal presentation of our model. As a first step, the results will be obtained in the context of a simple model with proportional tax rates, since this initial stage is sufficient for the purpose and also makes the relevant points as clear as possible. The complete model with progressive-regressive tax rates will be presented and analyzed immediately after the description of the simple model.

3.2. Asimple Model with Proportional Tax Rates

Although the nature of tax effort may seem very clear at an intuitive level, once a formalization is attempted, a number of issues arise that need to be resolved. Of most importance is the question of which form of utility function to choose or which constraints/postulates to adopt in order to evaluate an optimal level of tax revenue.[3] In experimenting with the appropriate utility maximization approach, we will start with a proportional tax system.

To provide a reasonably simple derivation of an optimal tax-revenue rule, we assume that the real disposable income, Y^d, takes the form:

$$Y^d = \frac{\left(1 - t_y\right)Y}{\left(1 + t_p\right)P} \tag{10}$$

where Y stands for GDP and P is the price level normalized at one. The interpretation of (10) is that real disposable income is favorably affected by a higher level of earned income but it is negatively associated with the indirect tax-induced increase in the price level. It is further assumed that the provision of public goods is financed by both an income tax, that bears a proportional tax rate (t_y) with respect to income, and a commodity tax with a proportional tax rate (t_ρ) with respect to consumption spending (C), that is

$$G = T = t_y Y + t_p C = T_y + T_\rho \tag{11}$$

Equation (11) is the standard government budget constraint that is used in the maximization process. The Lagrangean for the maximization problem can be written:

$$L = \ln\left(1 - t_y\right)Y - \ln\left(1 + t_p\right)P + \ln G + \lambda\left(G - t_y Y - t_p C\right) \tag{12}$$

The necessary conditions describing the choice of direct and commodity tax rates are:

$$\frac{dL}{dt_y} = -\frac{Y}{\left(1 - t_y\right)Y} - \lambda Y = 0 \Rightarrow -\frac{1}{1 - t_y} = \lambda Y \tag{13}$$

$$\frac{dL}{dt_p} = -\frac{P}{\left(1 - t_p\right)P} - \lambda Y = 0 \Rightarrow -\frac{1}{1 + t_P} = \lambda C \tag{14}$$

For the choice of the level of public good, optimizing with respect to G gives:

$$\frac{dL}{dG} = \frac{1}{G} + \lambda = 0 \Rightarrow \lambda = -\frac{1}{G} \tag{15}$$

Substituting (15) into (13) and (14) and rearranging gives:

$$G = \left(1 - t_y\right)Y = Y - T_y \tag{16}$$

$$G = \left(1 + t_p\right)C = C + T_P \tag{17}$$

Equating (16) with (17) and rearranging gives:

$$Y - T_y = C + T_i \Rightarrow Y - C = T_y + T_p$$

or

$$Y - C = T \tag{18}$$

The interpretation of (18) is that the optimal level of total taxation is equal to the difference between GDP and private consumption spending. This finding is set against the alternative view that the optimal or potential tax revenue is generated from the predicted (fitted) values based on a

3 The assumption made in Equation (3) is that the budget is balanced, that is the deficit is zero. Adopting the alternative assumption of a deficit budget does not affect the estimation results because, in deriving the first order conditions from the maximization process, the derivatives with respect to the exogenously given deficit are zero.

regression model, including a vector of other independent variables. As illustrated in Appendix B, these variables may be either measurable (demographic or macroeconomic variables such as the share of agriculture in GDP, the ratio of imports and exports to GDP, and so on) or unobserved (institutional, structural, administrative, ethical etc.). If this alternative view had been adopted, we would have to create a structural model, based on the past literature, and then introduce the determinants of Equation (18), that is GDP and consumption.

However, the results of mixing together the optimization procedure of the present paper and the conventional regression analysis would be highly controversial. Macroeconomic variables are constituent parts of GDP and all of the observed and unobserved determinants of tax effort that are employed in the standard relevant regressions have already left their own stamp on the way income and consumption have evolved. Combining determinants of GDP (consumption) with GDP (consumption) per se in order to construct a single tax effort relationship would possibly lead to serious multicollinearity problems, even if an assumption could be adopted that realistic effects would be produced by integrating optimization and standard regression techniques.

Given that the denominator of the tax effort index is determined by Equation (18) and the numerator is given by the sum of actual (direct and indirect) taxes, the relevant tax-effort outcome is shown in Table 4.

Obviously, the tax-effort index is derived from dividing the sum of actual (or realized) tax revenue, incorporating direct and indirect taxes, by the optimal level of taxation (18).

There are four important points that need to be made here:

(1) The actual tax burden over the period 1995–2009 is below its optimal level for each of the sample countries, with the exception of Croatia and Italy. The mean tax-effort index of all the sample countries (0.84) is either lower (in five cases) or higher (in one case) than its corresponding values obtained under the linear regression methodology in Table 2.

(2) In the six years following the eruption of the world financial (and debt) crisis, 2010–2015, nine out of the thirty sample countries raised their actual tax burden, whereas, in four out of these nine countries (Greece, Italy, Portugal, France), the actual tax burden exceeded the optimal one. Finally, it is noteworthy that, over the entire period 1995–2015, it is only two countries (Greece and Italy), which appear to overtax their taxpayers.

(3) Three EU countries (Greece, Cyprus, Portugal) having high debt/GDP ratios and running excessive government budget or balance-of-payments deficits in 2010–2015 had to increase their actual tax burden above (or close to) the optimal tax level. The same was true with Italy with a debt to GDP ratio far in excess of 100%. The first three countries, in addition to initiating high tax-effort indices, adopted strict stabilization programs monitored by the EU authorities and accompanied by financial aid on a massive scale.

(4) It would be advisable to enrich the discussion of the results by allowing for possible shortcomings of the developed model, especially with respect to tax-effort indices taking values less than one. For example, one might argue that the tax effort index falls short of one because of ignoring savings (or investment). In this case, overtaxing in Greece or Italy may be attributed to a drastic reduction in savings. Adequate domestic savings facilitate the financing of budget deficits from own country's sources and obviate the need to borrow money from abroad. Japan is an extreme case of a country with the highest debt/GDP ratio in the developed world, a large portion of which is held by Japanese investors. Greece is the completely different extreme case of a country with the second highest debt/GDP ratio and negative savings, while the government bonds are held by foreign public or private agents.

Despite the fact that the Arrow-Debreu model considers a fixed labor-supply economy with zero savings, we will make an attempt to slightly modify the original version by introducing a savings variable. In the present context, the real disposable income (10) is no longer spent entirely on the purchase of consumption goods, but a modest portion, equal to the difference between income and consumption, is saved:

$$Y^d = \frac{c\left(1-t_y\right)Y}{\left(1+t_p\right)p} + S \quad (19)$$

where $0 < c < 1$ stands for the average propensity to consume and S represents the size of personal savings.

The utility function is now described b

$$U = U\left(Y^d,\ G\right) = lnY^d + lnG\ = \ln\left[\frac{c\left(1-t_y\right)Y}{1+t_p} + S\right] + \ln\left[t_yY + \frac{Ct_p\left(1-t_y\right)Y}{1+t_p}\right] \quad (20)$$

The relationship (20) is then maximized by choice of direct and indirect tax rates and the first-order conditions are manipulated to give the optimal values of t_y and t_p,

$$t_y^* = \frac{(1-c)S + (7c-1) - 6c^2)Y - (c-1)\sqrt{S^2 - 14SY + Y^2}}{2[3Y(2c-1-c^2)]} \quad (21)$$

The optimal direct tax rate (21) in the savings-including model can be set against the corresponding rate (13) of the no-savings model, $t_y^* = 1 - \frac{G}{Y}$, just to point out the simplicity and the important predictive power of the conventional Arrow-Debreu model, employed in the present study.

The formula for the optimal indirect tax rate in the savings-including model is much more complicated than (21) and is available by the authors upon request.

Table 4. Tax effort index using the optimization process of the present study.

Country (1)	1995–2009 (2)	2010–2015 (3)	1995–2015 (4)
Belgium	0.95	0.97	0.96
Bulgaria	0.82	0.74	0.81
Czech Republic	0.66	0.66	0.66
Denmark	0.92	0.90	0.92
Germany	0.92	0.88	0.91
Ireland	0.59	0.49	0.55
Greece	0.97	1.25	1.04
Spain	0.82	0.80	0.81
France	0.98	1.05	1.00
Croatia	1.04	0.90	0.99
Italy	1.01	1.11	1.04
Cyprus	0.78	0.98	0.84
Latvia	0.77	0.75	0.76
Luxembourg	0.61	0.57	0.60
Hungary	0.83	0.79	0.82
The Netherlands	0.72	0.67	0.70
Austria	0.94	0.92	0.94
Poland	0.93	0.84	0.89
Portugal	0.93	1.05	0.96
Romania	0.93	0.73	0.89
Slovenia	0.83	0.83	0.83
Slovakia	0.73	0.69	0.72
Finland	0.86	0.94	0.88
Sweden	0.88	0.81	0.86
United Kingdom	0.99	1.00	1.00
Iceland	0.89	0.74	0.84
Norway	0.74	0.68	0.72
Switzerland	0.63	0.59	0.61
United States	0.80	0.80	0.80
Japan	0.59	0.69	0.62
Average	**0.84**	**0.83**	**0.83**

The correlation coefficients between the tax-effort index of our utility maximization model (column 2, Table 4) and the tax-effort indices of the regression analysis approaches (columns 1–5, Table 2) take on a wide range of values, as can be seen in Table 5[4].

- They are correlated in five out of the six cases, according to Spearman coefficient (column 3, Table 5).
- They are correlated in four out of the six cases according to Pearson (centered) coefficient (column 1, Table 5).
- They are correlated in all of the six cases, according to Pearson (uncentered) coefficient (column 2, Table 5).

According to these findings, it can be argued that our utility maximization method generates tax-effort indices which, in general, do not differ significantly from those in previous studies.

Note that, when dealing with (Pearson's) correlation coefficients, the value of 1 implies that a linear equation describes the relationship between two variables perfectly. A value of zero implies that there is not linear correlation between them.

In general, the interpretation of a correlation coefficient depends on the context and purposes of a particular study. For example, a correlation of 0.7 may be very low if one is verifying a physical law using high-quality instruments, but may be regarded as very high in the social sciences where there may be a greater contribution from complicating factors. Finally, it should be noted that statistical inference aims at testing the null hypothesis that the true sample correlation coefficient is equal to zero.

Table 5. Comparing the correlation coefficients.

	Present Study Utility Function (2)		
	Pearson Centerered (1)	**Pearson Uncentered (2)**	**Spearman Rank Order (3)**
IMF (standard regression)	0.4839 (0.0067)	0.9886 (0.0000)	0.4903 (0.0059)
World Bank (standard regression)	0.5678 (0.0011)	0.9863 (0.0000)	0.5799 (0.0008)
Stochastic frontier	0.5149 (0.0060)	0.9899 (0.0000)	0.4336 (0.0238)
Balanced Budget	0.0223 (0.9118)	0.9839 (0.0000)	0.0276 (0.8909)
Present study GMM (standard regression)	0.2600 (0.1652)	0.9175 (0.0000)	0.3439 (0.0627)
Present study GLS (standard regression)	0.3582 (0.0519)	0.9537 (0.0000)	0.3799 (0.0384)

Note: The critical values of the above correlation coefficients that must be exceeded to be considered significantly non-zero at the 0.06 level is 0.34 for our sample of $30 \times 14 = 420$ observations.

At first glance, Equation (18) appears to demonstrate how an optimal tax revenue indicator can be obtained in an economy with distortionary taxes by the use of a properly designed maximization technique. This is because manipulation of the first order conditions (13) and (14) succeed in maximizing social welfare by equating tax revenue to the difference between GDP and private consumption in

[4] In Table 5, the Pearson centered correlation coefficients refer to data which have been shifted by the sample means of their respective variables, so as to have an average of zero for each variable.

a way that previous literature on tax effort failed to do. This is an important observation that lends support to the argument of compatibility of the tax effort theory with optimizing behavior.

On the basis of the preceding discussion, one can possibly argue that the proposed new measure of tax effort fails to give rise to a stubborn conviction that it can avoid criticisms made of previous approaches or that it is more reliable or useful than existing measures. There may exist various reasons for giving our estimates a cautious welcome:

(i) Tax effort is treated as a latent variable. It might thus be appropriate to derive a range of estimates from different methods. The true value is likely to be within the range. However, if the width varies across countries, it is difficult to establish its location within this range.

(ii) The changes appearing when we compare the subperiods 2010–2015 and 1995–2009 offer a number of clues. It may be plausible that implied tax effort increases significantly in Greece, Italy and Portugal as these countries were severely hit by the financial crisis, but this does not explain the increase in France and, more importantly, it does not explain the decrease in Ireland or Germany. Thus, the paper seems, at first glance, to lack elaboration on the validity of the new estimates.

(iii) As shown in Table 4, it is only four out of thirty sample countries which are not below the estimated optimal level (period 1995–2015). This finding does not significantly differ from the estimates (see Table 2) of IMF, with all countries considered being below the estimated optimal level (over the period 1991–2012), from the estimates of the stochastic frontier approach with four countries above the optimal level, as well as from the estimates of the balanced budget index, with seven countries above the optimal level. It is only the World Bank, that presents a tax effort index in excess of unity for half of the countries examined (fifteen out of thirty) over the period 1994–2009.

With the above finding in mind, one might wonder whether our analysis would be improved by discussing the possibility that the estimated optimal level overstates what fiscal authorities may want to raise in tax revenue. For example, political economy considerations, such as implementing an expansionary fiscal policy in periods of recession, may motivate below optimal tax. This would provide an opportunity to discuss how factors excluded from the theoretical model may explain lower observed tax levels. In such a framework, it could be structural to trace out whether public borrowing would imply lower than optimal current tax.

Turning back to our initial modelling structure, let us assume that the basic Arrow-Debreu postulate of balanced budget can be recast in terms of a budget in deficit, in which public expenditure are financed by a combination of taxes and public borrowing, B. In this case, the government budget constraint (3) would take the following form,

$$G = t_y Y + t_p C + B$$

with the Lagrangean (12) being transformed to

$$L = \ln\left(1 - t_y\right)Y - \ln\left(1 + t_p\right)P + lnG + \lambda\left(G - t_y Y - t_p C - B\right)$$

Maximizing the last relationship with respect to t_y, t_p and G and solving the corresponding first-order conditions, we find the same equilibrium condition (18), that is

$$Y - C = T$$

Thus, in the Arrow-Debreu model of a closed economy, with the two structural postulates of no savings and fixed labor supply, the inclusion of deficit financing as a means of financing government spending does not affect the optimal level of tax revenue. The explanation lying behind this finding is that, in the absence of potential borrowing from abroad and with no domestic savings, the only possibility that the government could raise additional borrowing funds is to further increase the (direct-indirect) tax rates.

The foregoing analysis provides the motivation behind the potential decision of researchers to use the proposed approach. The benefit that the measure of optimal tax effort provided by the present model offers over and above the conventional tax effort measure is that the latter simply compares actual revenue to an estimate derived from notional capacity, based on social-political-economic features associated with determinants of revenue. In contrast, the approach suggested in the present text offers a different interpretation, in the sense that actual revenue compares to a theoretical optimal tax level, that takes into account only three crucial macroeconomic variables (GDP, tax revenue, consumption). These variables already embody by assumption, among others, the vast number of (observed and unobserved) socioeconomic and remaining factors employed by previous researchers.

It should be noted that intra-country (or inter-country) analysis of the factors which could explain movements in tax effort over a multi-year period (or across countries) would require more sophisticated theoretical and econometric methodological approaches. It would also have to extend the range of subjects that could be illustrated in the present study. However, the aim of our analysis is not to set our estimates of tax effort against the estimates of other researchers and trace out their differences; rather, it is to present a new optimization technique for evaluating tax effort by using observable macroeconomic variables alone, excluding at the same time other existing, but not yet very noticeable or well developed latent data as ad hoc regressors in tax equations.

3.3. A Complete Model with Progressive-Regressive Tax Rates

The simplicity of the preceding model with proportional taxation may raise doubts about the validity of our findings. In an attempt to move the theory closer to practical application, the explicit formulae for progressive income taxes and regressive commodity taxes described in Section 3.1 will be accounted for. Moreover, a modified utility function will be employed and a different methodology will be adopted, though the same statistical data will be used to verify (or nullify) the results of Table 4.

In this context, the utility function (2) will be slightly modified by replacing disposable income with private consumption,

$$U = U(C, G) = \ln C + \ln G \tag{22}$$

The net (of commodity taxes) consumption will be defined as:

$$C = \frac{c\left(1 - t_y\right)Y}{1 + t_p} = \frac{c(1 - \beta Y)Y}{1 + t_r \frac{Y^2}{\gamma}} \tag{23}$$

by using relationships (6), (7a), (7b), (8), (9a) and (9b).

The government budget constraint is given by the relation

$$T = T_y + T_p = t_y Y + t_p C = \beta Y^2 + \frac{t_r Y^2}{\gamma} C \tag{24}$$

Note that the personal income tax is recast in terms of a progressive tax and the commodity tax is also redefined to capture both its proportional (with respect to consumption) and regressive (with respect to income) features, constrained by the requirement that whatever definition is employed (proportional or regressive) will yield the same tax revenue, that is:

$$T_\rho = t_p C = t_r Y \tag{25}$$

The optimal allocation of resources is found from the Lagrangean

$$L = \ln[c(1 - \beta Y)Y] - \ln\left(1 + \frac{t_r Y^2}{\gamma}\right) + \ln G + \lambda\left[G - (\beta Y)Y - \frac{t_r Y^2 C}{\gamma}\right] \tag{26}$$

Carrying out the maximization in (26) provides the efficiency criteria

$$\frac{dL}{d\beta} = -\frac{cY^2}{c(1-\beta Y)Y} - \lambda Y^2 = 0 \Rightarrow G = \left(1 - t_y\right)Y \tag{27}$$

$$\frac{dL}{d\gamma} = -\frac{\frac{t_r Y^2}{\gamma^2}}{1 + \frac{t_r Y^2}{\gamma}} + \lambda \frac{t_r Y^2 C}{\gamma^2} = 0 \Rightarrow G = \left(1 + \frac{t_r}{\gamma} Y^2\right)C = \left(1 + t_p\right)C \tag{28}$$

$$\frac{dL}{dG} = \frac{1}{G} + \lambda = 0 \Rightarrow \lambda = -\frac{1}{G} \tag{29}$$

Equating the first two efficiency conditions and rearranging gives:

$$Y - C = t_y Y + t_\rho C$$

or

$$Y - C = T \tag{30}$$

It is worth noting that the optimal level of tax revenue has been found to be equal to the difference between GDP and private consumption (net of indirect taxes), as it was also the case with the simple model of proportional taxation in Section 3.2. Consequently, the conclusions which have been drawn from the estimation of the optimal tax revenue, on the basis of the model with proportional tax rates in Section 3.2, are also shown to hold for the model with progressive/regressive tax rates in this section.

It should be stressed, however, that both models, with (non)proportional tax rates, only provide an implicit expression for the optimal tax revenue and precise statements cannot be made without further restrictions. This observation does not mean that some general comments cannot be made. Accepting the approximation interpretation, this suggests that the calculation of optimal tax revenue does not necessarily lead to an optimal distribution of this revenue to the various types of taxes (in our paradigm, direct and indirect ones). Although broadly correct, this statement can only be truly justified when the policy makers' decisions, regarding the allocation of tax receipts, are accounted for. One simple case that overcomes this difficulty is that in which there are no equity or efficiency effects between the tax categories or that these effects offset each other. This limiting case has been considered in the present paper.

Returning to the general case and putting the incentive/disincentive aspects of various taxes into practice, the distortionary structure of taxation could involve low-income households paying disproportionately larger fractions of their income in indirect taxes or high-income households paying disproportionately larger fractions of their income in direct taxes.[5] The potentially inequitable or inefficient nature of this outcome simply reflects the absence of a parameter that would measure the government's volition to optimally allocate the tax receipts to various categories. The objective function of our model, however, appears to ignore equity or efficiency criteria and the solution reflects only revenue-sufficiency considerations.

In summary, the equilibrium determined by setting the optimal tax revenue equal to actual tax revenue (i.e., a unitary tax-effort index) is second best compared to the outcome that would arise if the process of estimating the optimal tax revenue would be combined with a process of describing an optimal distribution of the (optimal) tax revenue to direct and indirect taxes. An attempt to find such a combination can provide a scope for further research in the field of optimal taxation. For the

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5 In the context of the present analysis, some tax rates might show up even with zero or unit values. The interpretation that could be given is that the Lagrangeans (12) and (26) are maximized with respect to tax rates while the corresponding utilities are adversely affected by taxes (downward sloping utility curves). To avoid extreme values in tax rates, our model should be extended to include parameters justifying government intervention in the area of optimal allocation of (the optimal) revenue to (in)direct taxes, something which is beyond the scope of the present study.

time being, the substitution effects, which distort the household's optimal choices and are unavoidable when (in)direct taxes are employed, are postulated to lead to efficiency losses, which can be minimized by adopting the optimal taxation rule described in Section 3.

4. Summary and Conclusions

The fundamental conclusion of this paper is that a conditionally distortionary tax system can lead to a Pareto optimal tax revenue and, hence, to a Pareto optimal tax effort index that is argued to be more reliable than the relevant index that has been estimated by previous contributions to this issue. Of most importance is our finding that the optimal level of total taxation is equal to the difference between GDP and private consumption. On the basis of this definition, the actual tax burden has been estimated to be below its optimal level for the majority of the countries studied.

One might argue that our model developed in Section 3 is over-simplified and inadequate to determine the optimal level of taxation because the results are an artifact of its very restrictive and rather unrealistic assumptions. Such assumptions (in addition to those listed above) may possibly be summarized as follows: (i) we impose a separable utility function that depends only on private consumption and public goods and no room is left for intertemporal utility maximization, (ii) there is no option to save and invest or to build up a production function that interacts with the consumption bundle, so that the economy appears to operate in an endowment (Y) model, (iii) our analysis fails to focus on the empirical part of Section 2, that has the potential for contribution under a well-specified structural/statistical model.

Due to all these assumptions, one could claim that our finding that optimal taxation depends only on the difference between GDP and consumption is simplistic and potentially misleading, since it ignores the vast literature on optimal taxation with important contributions on the field.

Such criticisms could typically be justified if we had selected any other theoretical/empirical framework lying beyond the frontiers set by the Arrow-Debreu economy, on which our model in Section 3 is founded. The Arrow-Debreu's competitive general equilibrium theory, in its simple form, postulates an economy with fixed labor supply (with labor being the only input in the production process and treated as numeraire with a constant wage rate), no savings, constant returns to scale, an initial endowment equal to the stock of labor services, separable utility functions, balanced government budget and so on (see Myles 1997, chp. 2).

Therefore, in the Arrow-Debreu environment, the combination of the household's disposable income constraint,

$$Y - T_y = C + T_p$$

and the government budget constraint (Equation (3))

$$G = T = T_y + T_p$$

gives

$$Y - C = T_y + T_p = T = G$$

which is the same result as the one derived from the maximization procedure of Section 3 (see Equation (18)).

Therefore, what is erroneously considered to be a simplistic and misleading model, i.e., the model in Section 3, is shown to verify the practical inference that the equilibrium tax revenue equals the difference between GDP and consumption. In addition, since the results of our maximization process are close to those of IMF and World Bank (see Table 5), we can ignore the previous conventional literature on optimal taxation and the relevant contributions based on complicated ad hoc relationships that use a large array of observed and unobserved explanatory variables (see discussion in Appendix A). Obviously, the present study provides a scope for researchers to extend the simple maximization model, described in Section 3, in the direction of employing advanced optimization approaches based on a variety of observed macroeconomic factors alone.

The results presented in the present study have a particular strong policy implication for all countries. It is presumably quite feasible for fiscal authorities to design and implement their tax schemes on the basis of the agents' potential tax capacity defined by the difference between GDP and consumption spending. The alternative view of making optimal tax burden to depend on a large number of possible determinants, based on governance structure, institutions, corruption and bureaucracy indices or on an array of economic, political or social variables, cannot take a robust analysis too far. Furthermore, improving all the above institutional and non-institutional factors is not necessarily easier than determining optimal tax performance by aggregating all the factors into two variables (income and consumption). In other words, the methodology proposed in our model requires only that the government is able to observe two fundamental macroeconomic variables from National Accounts, which is a far weaker informational requirement than collecting a lot of other unobservable variables.

There is an important policy lesson that can be drawn from this study. Our approach allows fiscal authorities to carefully monitor the evolution of government revenue (and, hence, expenditure), without having to depend on predicted variables, especially those involving institutional and political factors. Many countries tend to scale up public spending to meet infrastructure and social needs while, at the same time, focusing on administrative, institutional and political improvements to collect increased tax revenue. In this case, fiscal sustainability may be adversely affected in the absence of a sound fiscal framework, which would allow for a robust-estimate of optimal tax revenue.

Author Contributions: B.D. developed the theoretical formation, conceived and planned the experiments, designed the model and the computational framework, devised the project and the conceptual ideas, analyzed the results and wrote the manuscript. P.P. performed the computations and calculations, carried out the experiments, contributed to the sample preparation, and assisted with S.T. measurements and worked out some of the technical details. S.T. verified the analytical methods, investigated special aspects, contributed to the interpretation of the results, analyzed the data and carried out the implementation, verified the numerical results of B.D, processed the experimental data with P.P., discussed the results and commented on the manuscript with B.D.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

Appendix A. Approaches to Calculating Tax Effort

The four approaches which are used in existing literature to determine tax effort differ fundamentally by the way in which the key variable of potential tax revenue is calculated.

Appendix A.1. The Standard Regression Approach

In this approach, tax effort is measured as the ratio of actual tax collection to the potential tax revenue. The potential tax revenue is, in turn, generated from the predicted (fitted) values based on regression analysis. Two representative contributions to this approach are summarized below:

(1) Lee, Dodson and Bayraktar of the World Bank (2012), using a sample of 110 countries, have found that, when the institutional variables (governance quality, i.e., bureaucracy quality and corruption) are included with income, income loses its significance. The demographic characteristics (population growth, age dependency) and the macroeconomic variables (trade openness, agricultural value added) are statistically significant for the whole period but not for some subperiods.

(2) Crivelly and Gupta of IMF (2014) constructed a database for 35 resource-rich countries over the period 1992–2009 and found support for the hypothesis that a large offset in tax (non resource) revenue is associated with an increased share of resource revenues. In addition, tax revenue has shown to depend negatively on the share of agriculture in GDP and foreign aid, positively on development, inflation and foreign indebtedness and both (positively or negatively) on trade openness, whereas no evidence of any relation between tax effort and corruption was detected.

Some early contributions to this discussion also were those of Bahl (1971); Lotz and Mors (1970); Leuthold (1991); Tanzi (1992); Stotsky and WoldeMariam (1997); Ghura (1998); Piancastelli (2001); Eltony (2002) and Gupta (2007). Two versions of the standard regression approach, but of limited reliability, are similarly mentioned in the literature: The income approach, where tax effort is defined as the ratio of actual tax collection to GDP, and the representative tax system (RTS) approach, which requires the identification of close proxies for the tax base corresponding to each tax category (Piancastelli 2001; Purohit 2006).

The basic criticisms of the standard regression approach can be summarized in two propositions:

(1) There is inadequate *a priori* justification for the use of the selected variables as measures of taxable capacity. Moreover, it is far from clear that this concept can be measured in any meaningful sense. For example, the complex problem of the relation between tax revenue and government spending is only one of the issues which are obscure in tax capacity exercises.

(2) The data employed in these studies are not reliable, even for the most advanced economies. No one can seriously claim that the estimates of corruption, bureaucracy, and the features of political systems or institutions are unquestionable. Furthermore, nobody can truthfully argue that he is aware of all the biases these estimates introduce to the results.

Appendix A.2. Stochastic Frontier Approach

Fenochietto and Pessino of IMF (2013) estimate tax capacity and tax effort for 113 countries, using a stochastic tax frontier model for panel data. They find that development, trade, education, inflation, income distribution and corruption are the central factors on which tax capacity depends.

Stochastic frontier analysis (SFA) to compute tax effort has been adopted by other researchers, too. See, for example, Alfirman (2003); Pessino and Fenochietto (2010); Garg et al. (2017); Langford and Ohlenburg (2016) and Brun and Diakite (2016). The SFA makes a distinction between the tax capacity of the standard regression analysis (i.e., maximum tax revenue that could be collected in a country, given its economic, social, institutional and demographic characteristics) and the potential tax collection (i.e., maximum revenue that could be obtained given the provisions of the prevailing tax system)[6]. Following this distinction, a line is drawn between the concept of tax effort (i.e., the ratio of actual tax revenue to tax capacity) and the concept of tax efficiency or tax gap (i.e., the difference between the potential tax collection and the actual tax revenue, which is a function of tax capacity). Thus, a country may have a low level of tax effort but a high level of tax efficiency, just because the government (majority of voters) chooses to levy lower taxes and to provide a low level of public goods.

The tax stochastic frontier model, which measures time-varying inefficiency, has two disturbances. The first disturbance is the usual mean zero statistical error term and the second one is the estimate of technical inefficiency. Another feature of the tax stochastic frontier model is the following: While some of the inputs used to produce the output (that is the potential tax revenue) are quite clear, such as openness, per capita GDP and level of education, other inputs, such as GINI coefficient and ease of tax collection, are not based on reliable estimates.

As becomes evident, the stochastic frontier tax function is an extension of the standard regression model, based on the theoretical premise that the production possibility frontier of taxation represents the maximum level of revenue that the government can achieve, considering a set of economic factors and revenue potential variables. As a consequence, it should be expected that the estimated tax effort from the standard approach and the stochastic frontier analysis will be highly correlated and the two methods will be quite substitutable. Such being the case, the advantage of the SFA may simply lie in having a more transparent interpretation of specific institutional constraints to tax effort.

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6 The stochastic frontier analysis is conducted in two stages. In the first stage, the SFA is used to model tax effort, while, in the second stage, factors influencing the time-varying inefficiency in tax effort are identified. Such factors include, in addition to the aforementioned ones, tax rates, exemptions and other elements of the tax structure, corruption and evasion, agriculture, capital investment, foreign grants, population density and so on.

Appendix A.3. The Budget Balance Approach

A third approach recently advanced by Cyan et al. (2013) is to measure the tax effort as the ratio of the desired level of taxation, which coincides with the preferred level of public expenditure, to the current tax revenue. The preferred level of public spending is considered to be the persistent or structural or long-run level of government expenditure. Equivalently, fiscal deficit is deemed to be the proper benchmark for capturing the discrepancy between desired tax revenue and current tax collections. The budget balance approach is quite simple, since it does not involve any commitment to running regressions or employing any other econometric techniques. However, it can hardly be recommended as a measure of evaluating tax effort, unless the direction of causality between revenue and expenditure is fully clarified.

Appendix A.4. The Welfare Maximization Approach

This approach attempts to account for some of the criticisms leveled at the three aforementioned estimated methods, by offering a stronger theoretical basis for the tax ratio. To this end, an optimal tax share is derived, assuming welfare maximizing behavior on the part of fiscal authorities, subject to the constraint of a balanced budget. The arguments of the welfare function to be maximized are private disposable income and public goods. The solution of the welfare maximization problem results in the desired tax share equation. The actual tax share is then taken to be a function of both the desired tax share and the availability of certain tax bases, such as the shares of agriculture, mining, exports and imports in income (Leuthold 1991) or, in addition, the status of economic policies and the level of corruption (Ghura 1998)[7]. Finally, the augmented actual tax share equation is treated as the basic model in these studies. The tax effort index is computed by taking the ratio of the actual (augmented) tax share to the predicted tax share. Thus, the welfare maximization approach appears to collapse to the standard regression approach, as the maximization process is not properly manipulated to provide the optimal level of tax effort.

Appendix B

Data Sources (time period: 1996–2015).

Study Variable	Dataset Variable	Source
Tax revenue (% of GDP)	Total tax burden including imputed social contributions, total economy	Macro-economic database AMECO, European Commission's Directorate General for Economic and Financial Affairs
Inflation rate	Consumer price index	Macro-economic database AMECO, European Commission's Directorate General for Economic and Financial Affairs
Fiscal deficit	General government net lending	Macro-economic database AMECO, European Commission's Directorate General for Economic and Financial Affairs
Growth rate	Gross domestic product at 2010 reference levels (OVGD)	Macro-economic database AMECO, European Commission's Directorate General for Economic and Financial Affairs
Indirect taxes	Taxes linked to imports and production (indirect taxes): general government: - ESA 2010 (UTVG)	Macro-economic database AMECO, European Commission's Directorate General for Economic and Financial Affairs

7 Ghura, in an analysis of data for 39 sub Saharan African countries, shows that variations in the tax revenue-GDP ratios are influenced positively by structural reforms, human capital, income, openness and inflation, but negatively by corruption and the share of agriculture in GDP.

Study Variable	Dataset Variable	Source
Direct taxes	Current taxes on income and wealth (direct taxes): general government: - ESA 2010 (UTYG)	Macro-economic database AMECO, European Commission's Directorate General for Economic and Financial Affairs
Investments	Gross fixed capital formation at 2010 prices: total economy (OIGT)	Macro-economic database AMECO, European Commission's Directorate General for Economic and Financial Affairs
Total private consumption	Private final consumption expenditure at 2010 prices (OCPH)	Macro-economic database AMECO, European Commission's Directorate General for Economic and Financial Affairs
Gross disposable income	Gross national disposable income (UVGT)	Macro-economic database AMECO, European Commission's Directorate General for Economic and Financial Affairs
External indebtedness	General government consolidated gross debt: - Excessive deficit procedure (based on ESA 2010)	Macro-economic database AMECO, European Commission's Directorate General for Economic and Financial Affairs
GDP per capita	GDP per head (US$ constant prices, reference year 2010)	OECD Database
Age dependency ratio (% of working-age population)	Age dependency ratio (in %) is the ratio of dependents—people younger than 15 or older than 64—to the working-age population—those ages 15–64.	OECD Database
Imports of goods and services (% of GDP)	Balance of Payments BPM6: Good imports & Services imports	OECD Database
Agriculture, value added (% of GDP)	Value added and its components by activity, ISIC rev4. VA0: Agriculture, forestry and fishing	OECD Database
Corruption	Reflects perceptions of the extent to which public power is exercised for private gain, including both petty and grand forms of corruption, as well as "capture" of the state by elites and private interests	Worldwide Governance Indicators, World Bank
Political Stability and Absence of Violence/Terrorism	Reflects perceptions of the likelihood that the government will be destabilized or overthrown by unconstitutional or violent means, including politically-motivated violence and terrorism.	Worldwide Governance Indicators, World Bank
Share of mining	Value added and its components by activity, ISIC rev4. VB: Mining and quarrying	OECD Database
Bureaucracy	Government Effectiveness: Reflects perceptions of the quality of public services, the quality of the civil service and the degree of its independence from political pressures, the quality of policy formulation and implementation, and the credibility of the government's commitment to such policies.	Worldwide Governance Indicators, World Bank.

Study Variable	Dataset Variable	Source
Foreign aid in relation to GDP	Government expenditure by function (COFOG), GOVEXP-Government expenditure by function-0102:Foreign Economic aid	OECD Database
Shadow economy as a percentage of GDP	Size of the shadow economy (% GDP)	1. Schneider and Williams (2013), The Shadow Economy, The Institute of Economic Affairs, London 2. Schneider (2015), Size and Development of the Shadow Economy of 31 European and 5 other OECD Countries from 2003 to 2015: Different Developments, Department of Economics, Johannes Kepler University Working Paper

References

Alesina, Alberto, and Allan Drazen. 1991. Why are stabilizations Delayed? *American Economic Review* 81: 1170–88.

Alfirman, Luky. 2003. Estimating Stochastic Frontier Tax Potential: Can Indonesian Local Governments Increase Tax Revenues under Decentralization? Working Paper No. 03-19, Center for Economic Analysis, Department of Economics, University of Colorado Boulder, Boulder, CO, USA.

Arrow, Kenneth, and Gerard Debreu. 1954. Existence of an Equilibrium for a Competitive Economy. *Econometrica* 22: 265–90. [CrossRef]

Bahl, Roy. 1971. A regression Approach to Tax Effort and Tax Ratio Analysis. *IMF Staff Papers* 18: 570–612. [CrossRef]

Brun, Jean, and Maïmouna Diakite. 2016. Tax Potential and Tax Effort: An Empirical Estimation for Non-resource Tax Revenue and VAT's Revenue', Études et Documents, n° 10, CERDI. Available online: http://cerdi.org/production/show/id/1814/type_production_id/1 (accessed on 10 July 2019).

Chelliah, Raja, Baas Hessel, and Margaret Kelly. 1975. Tax Ratios and Tax Effort in Developing Countries, 1969–1971. *IMF Economic Review* 22: 187–205. [CrossRef]

Crivelli, Ernesto, and Sanjeev Gupta. 2014. Resource Blessing, Revenue Curse? Domestic Revenue Effort in Resource-Rich Countries. IMF Working Paper, WP/14/5, International Monetary Fund, Washington, DC, USA.

Cukierman, Alex, Edwards Sebastian, and Guido Tabellini. 1992. Seigniorage and Political Instability. *The American Economic Review* 82: 537–55.

Cyan, Musharraf, Martiez-Vazquez Jorge, and Violeta Vulovic. 2013. *Measuring Tax Effort: Does the Estimation Approach Matter and Should Effort be Linked to Expenditure Goals?* Working Paper Series 39. Atlanta: International Center for Public Policy, Andrew Young School of Policy Studies, Georgia State University.

Dollery, Brian, and Andrew Worthington. 1996. The Empirical Analysis of Fiscal Illusion. *Journal of Economic Surveys* 10: 261–97. [CrossRef]

Edwards, Sebastian, and Guido Tabellini. 1991. Explaining Fiscal Policies and Inflation in Developing Countries. *Journal of International Money and Finance* 10: 835–57. [CrossRef]

Eltony, Nagy. 2002. Measuring Tax Effort in Arab Countries. Economic Research Forum Working Papers No. 0229, Arab Planning Institute, Kwait.

Fenochietto, Ricardo, and Carola Pessino. 2013. Understanding Countries' Tax Effort, IMF Working Paper, WP/13/244, International Monetary Fund, Washington, DC, USA.

Garg, Sandhya, Goyal Ashima, and Rupayan Pal. 2017. Why Tax Effort Falls Short of Tax Capacity in Indian States. *Public Finance Review* 45: 232–59. [CrossRef]

Ghura, Dhaneshwar. 1998. Tax Revenue in Sub-Saharan Africa: Effects of Economic Policies and Corruption. IMF Working Paper, WP/98/135, International Monetary Fund, Washington, DC, USA.

Gupta, Abhijit. 2007. Determinants of Tax Revenue Effects in Developing Countries. IMF Working Paper, WP/07/184, International Monetary Fund, Washington, DC, USA.

Kim, Sangheon. 2007. A more accurate measurement of tax effort. *Applied Economic Letters* 14: 539–43. [CrossRef]

Kmenta, Jan. 1986. *Elements of Econometrics*, 2nd ed. New York: Macmillan Publishing.

Langford, Ben, and Tim Ohlenburg. 2016. Tax Revenue Potential and Effort. Working Paper, S-43202-UGA-1, International Growth Centre, London School of Economic and Political Science, London, UK.

Le, Tuan Minh, Blanca Moreno-Dodson, and Nihal Bayraktar. 2012. Tax Capacity and Tax Effort: Extended Cross-Country Analysis from 1994 to 2009. World Bank Policy Research Working Paper No. 6252, The World Bank, Washington, DC, USA.

Leuthold, Jane. 1991. Tax Shares in Developing Economies: A Panel Study. *Journal of Development Economics* 25: 173–85. [CrossRef]

Lotz, Joergen, and Elliott Mors. 1967. Measuring Tax Effort in Developing Countries. *IMF Staff Papers* 14: 478–99. [CrossRef]

Lotz, Joergen, and Elliott Mors. 1970. A Theory of Tax Level Determinants for Developing Countries. *Economic Development and Cultural Change* 18: 328–41. [CrossRef]

Myles, G. 1997. *Public Economics*. Cambridge: Cambridge University Press.

Oates, Wallace W. 1988. On the Nature and Measurement of Fiscal Illusion: A Survey. In *Taxation and Fiscal Federalism: Essays in Honour of Russell Mathews*. Edited by Geoffrey Brennan, Bhajan S. Grewal and Peter Groenewegen. Sydney: ANU Press.

Pessino, Carola, and Ricardo Fenochietto. 2010. Determining countries' tax effort. *Hacienda Pública Española* 195: 65–87.

Piancastelli, Marcelo. 2001. Measuring the Tax Effort of Developed and Developing Countries: Cross Country Panel Data Analysis—1985/95. IPEA Working Paper, no. 818, Institute for Applied Economic Research, Rio de Janeiro, Brazil.

Przeworski, Adam, and Fernando Limongi. 1993. Political Regimes and Economic Growth. *The Journal of Economic Perspectives* 7: 51–69. [CrossRef]

Purohit, Mahesh. 2006. Tax Efforts and Taxable Capacity of Central and State Government. *Economic and Political Weekly* 25: 747–55.

Schneider, Friedrich. 2015. Size and Development of the Shadow Economy of 31 European and 5 other OECD Countries from 2003 to 2015: Different Developments. Working Paper, Department of Economics, Johannes Kepler University, Linz, Austria.

Schneider, Friedrich, and Colin Williams. 2013. *The Shadow Economy*. London: The Institute of Economic Affairs.

Stotsky, Janet Gale, and Asegedech WoldeMariam. 1997. Tax Effort in sub-Saharan Africa. IMF Working Paper, WP/97/107, International Monetary Fund, Washington, DC, USA.

Tait, Alan, Grätz Wilfrid, and Barry Eichengreen. 1979. International Comparisons of Taxation for Selected Developing Countries, 1972–1976. *IMF Staff Papers* 26: 123–56. [CrossRef]

Tanzi, Vito. 1992. Structural factors and tax revenue in developing countries: A decade of evidence. In *Open Economies: Structural Adjustment and Agriculture*. Edited by Ian Goldin and Alan Winters. New York: Cambridge University Press.