Archives of Current Research International
4(2): 1-13, 2016, Article no.ACRI.25971
ISSN: 2454-7077

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Variable Heat and Mass Transfer Boundary on an Unsteady MHD Flow through a Porous Medium over an Infinite Vertical Plate

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

Article Information

DOI:10.9734/ACRI/2016/25971
Editor(s):
(1) Preecha Yupapin, Department of Physics, King Mongkut's Institute of Technology Ladkrabang, Thailand.
Reviewers:
(1) Islam M. Eldesoky, Menofia University, Egypt.
(2) John Abraham, University of St. Thomas, USA.
(3) Promise Mebine, Niger Delta University, Nigeria.
Complete Peer review History: http://sciencedomain.org/review-history/14596

Original Research Article

Received 28th March 2016
Accepted 4th May 2016
Published 12th May 2016

ABSTRACT

We have considered the variable heat and mass transfer boundary on an unsteady MHD flow through a loosely packed porous medium over an impulsively started vertical plate. The temperature of plate is made to rise linearly with time. The fluid considered is gray, absorbing-emitting radiation but a non-scattering medium. The governing equations involved in the present analysis are solved by the Laplace-transform technique. The velocity, skin friction, Nusselt number and Sherwood number are obtained and computationally discussed for different governing parameters such as radiation parameter, Schmidt number, Thermal Grashof number, mass Grashof number, magnetic field parameter, porous parameter and Prandtl number with the combination of the other flow parameters are illustrated graphically, and physical aspects of the problem are discussed.

Keywords: Radiation effects; MHD flows; heat and mass transfer boundary; vertical plates; porous medium; skin friction; nusselt number; sherwood number.

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NOMENCLATURES

β	: Volumetric coefficient of thermal expansion
β *	: Volumetric coefficient of expansion with concentration
σ	: Stefan–Boltzmann constant
ρ	: Density
θ	: Dimensionless temperature
ν	: Kinematic viscosity
μ	: Coefficient of viscosity
τ	: Dimensionless skin friction
b	: Similarity parameter
a^*	: Absorption coefficient
A	: Constant
B_0	: External magnetic field
C	: Species concentration in the fluid
$\bar{C}$	: Dimensionless concentration
C_p	: Specific heat at constant pressure
C_w	: Concentration of the fluid
C_∞	: Concentration in the fluid far away from the plate
D_1	: Chemical molecular diffusivity
erf	: Error function
erfc	: Complementary error function
g	: Acceleration due to gravity
Gm	: Mass Grashof number
Gr	: Thermal Grashof number
D	: Darcy parameter
k	: Thermal conductivity of the fluid
M	: Magnetic field parameter
Nu	: Dimensional Nusselt number
Pr	: Prandtl number
q_r	: Radiative heat flux in the y direction
R	: Radiation parameter
Sc	: Schmidt number
Sh	: Dimensional Sherwood number
T	: Temperature of fluid near the plate
t	: Time
$\bar{t}$	: Dimensional time
T_w	: Temperature of the fluid
T_∞	: Temperature of the fluid far away from the plate
u	: Velocity of the fluid in the x - direction
u_0	: Velocity of the fluid
$\bar{u}$	: Dimensionless velocity
y	: Coordinate axis normal to the plate
$\bar{y}$	: Dimensionless coordinate axis normal to the plate

SUBSCRIPTS

w, 0	Conditions on the wall
∞	Free stream conditions

1. INTRODUCTION

Many transport processes exist in nature and industrial applications in which the transfer of heat and mass occurs simultaneously as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species. In the last few decades several efforts have been made to solve the problems on heat and mass transfer in view of their application to astrophysics, geophysics and engineering. The study of MHD flow with heat and mass transfer plays an important role in biological Sciences. Effects of various parameters on human body can be studied and appropriate suggestions can be given to the persons working in hazardous areas having noticeable effects of magnetism and heat variation. Study of MHD flows also has many other important technological and geothermal applications. Major important applications are cooling of nuclear reactors, liquid metals fluid, power generation system and aero dynamics. The effects of radiation on free convection on the accelerated flow of a viscous incompressible fluid past an infinite vertical porous plate with suction has many important technological applications in the astrophysical, geophysical and engineering problem. Free convection effects on the oscillating flow past an infinite vertical porous plate with constant suction was studied by Soundalgekar [1] which was further improved by Vajravelu and Sastri [2]. Soundalgekar and Takhar [3] studied the MHD flow and heat transfer over a semi-infinite plate under the influence of uniform transverse magnetic field. Also Soundalgekar and Wavre [4] have studied unsteady free convection flow past an infinite vertical plate with variable suction and mass transfer. Soundalgekar and Takhar [5] have considered radiation effects on free convection flow past a semi-infinite vertical plate. Das et al. [6] have studied effects of mass transfer on flow past an impulsively started vertical infinite plate with constant heat flux and chemical reaction. Ezzat Magdey [7] has considered magneto hydro dynamic unsteady flow of non-Newtonian fluid past an infinite porous plate. Radiation and free convection flow past a moving plate was considered by Raptis and Perdikis [8]. Muthucumara swamy et al. [9] analyzed theoretical solution of flow past an impulsively started vertical plate with variable temperature and mass diffusion. Jaiswal and Soundalgekar [10] have considered the oscillating plate temperature effects on a flow past an infinite porous plate with constant suction and embedded in a porous medium. Muthucumara swamy and Ganesan [11] have studied heat transfer effects on flow past an impulsively started semi-infinite vertical plate with uniform heat flux. Magyari et al. [12] have studied vertical flat plate embedded in a stably stratified fluid saturated porous medium. Prasad et al. [13] studied effects of heat and mass transfer on flow past an oscillating vertical plate with variable temperature. Muthucumara swamy et al. [14] studied unsteady flow past an accelerated infinite vertical plate with variable temperature and uniform mass diffusion. Toki [15] improved the analytical solutions for free convection and mass transfer flow near a moving vertical porous plate. Thermal radiation effect on a transient MHD flow with mass transfer past an impulsively fixed infinite vertical plate was studied by Ahmed and Sarmah [16]. Reddy and Reddy [17] investigated Soret and Dufour effects on steady MHD free convection flow past a semi-infinite moving vertical plate in a porous medium with viscous dissipation. Das [18] developed the closed form solutions for the unsteady MHD free convection flow with thermal radiation and mass transfer over a moving vertical plate. In this continuation, the effect of heat and mass transfer on unsteady MHD free convection flow past a moving vertical plate in a porous medium was investigated by Das and Jana [19]. Prasad et al. [20] discussed radiation and mass transfer effects on unsteady MHD free convection flow past a vertical porous plate embedded in a porous medium. Rajesh [21] have considered MHD effects on free convection and mass transform flow through a porous medium with variable temperature. Osman et al. [22] discussed the thermal radiation and chemical reaction effects on unsteady MHD free convection flow through a porous plate embedded in a porous medium with heat source/sink and the closed form solutions are obtained. Seth et al. [23] investigated the unsteady MHD natural convection flow with radiative heat transfer past an impulsively moving plate with ramped wall temperature. Srinivasacharya and Kaladhar [24] discussed the Soret and Dufour effects on the mixed convection heat and mass transfer along a semi-infinite vertical plate embedded in a non-Darcy porous medium saturated with couple stress fluid. The combined effects of heat and mass transfer on free convection unsteady magneto hydrodynamic flow of viscous fluid embedded in a porous medium is discussed by Farhad Ali et al. [25]. Rao and Krishna [26] discussed the combined effects of radiative heat transfer and a transverse magnetic field on steady rotating flow of an electrically conducting optically thin fluid through a porous medium in a parallel plate channel and non-uniform temperatures at the walls. Recently, unsteady flow of a viscous incompressible fluid past an exponentially accelerated moving vertical plate had been investigated by Ahmed et al. [27]. An unsteady hydro magnetic flow of a viscous incompressible electrically conducting fluid past an accelerated porous flat plate in the presence of a uniform transverse magnetic field in a rotating system taking the hall effects into account have been discussed by Das et al. [28]. The radiation effect on the thermo-magnetic convection which occurs in participating paramagnetic medium under microgravity condition is numerically investigated by Wang and Tan [29]. The thermal-diffusion and diffusion-thermo effects on heat and mass transfer by transient free convection flow of over an impulsively started isothermal vertical plate embedded in a saturated porous medium were numerically investigated by EL-Kabeir et al. [30]. Ramesh and Devakar [31] studied the influence of heat transfer on the peristaltic transport of an incompressible magneto hydro dynamic second grade fluid in vertical symmetric and asymmetric channels. Khamisah Jafer et al. [32] discussed the steady magnetohydrodynamic (MHD) laminar boundary layer flow of a viscous and incompressible electrically conducting fluid near the stagnation point on a horizontal stretching or shrinking surface, with variable surface temperature and a constant magnetic field applied normal to the surface of the sheet. Ali et al. [33] discussed the steady magneto hydrodynamic mixed convection boundary layer flow of an incompressible, viscous and electrically conducting fluid over a stretching vertical flat plate is theoretically investigated with Hall effects taken into account. The problem of laminar fluid flow which results from the simultaneous motions of a freestream and its bounding surface in the same direction has been investigated numerically by Abraham and Sparrow [34]. In this paper, we have considered radiation effects on MHD flow past an impulsively started vertical plate with variable heat and mass transfer boundary. The results are shown with the help of tables and graphs.

2. FORMULATION AND SOLUTION OF THE PROBLEM

We consider the flow of unsteady viscous incompressible fluid through a porous medium past a vertical plate. The *x*- axis is taken along the plate in the upward direction and *y*-axis is taken normal to the plate. Initially the fluid and plate are at the same temperature. A transverse magnetic field B_0 of uniform strength is applied normal to the plate as shown in Fig. 1. The viscous dissipation and induced magnetic field has been neglected due to its small effect. Initially, the fluid and plate are at the same temperature T_∞ and concentration C_∞ in the stationary condition. At time t >0, the plate is moving with a velocity $u = u_0$ in its own plane and the temperature of the plate is raised to T_w and the concentration level near the plate is raised linearly with respect to time.

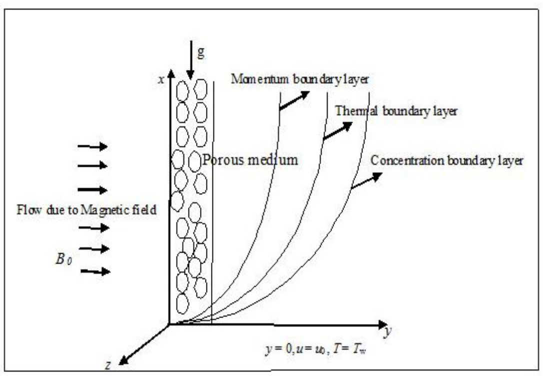

Fig. 1. Physical configuration of the problem

The unsteady equations of the MHD flow through porous medium are as:

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_e B_0^2}{\rho} u - \frac{\nu}{K} u + g\beta(T - T_\infty) + g\beta^*(C - C_\infty) \quad (1)$$

$$\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \quad (2)$$

$$\frac{\partial C}{\partial t} = D_1 \frac{\partial^2 C}{\partial y^2} \quad (3)$$

The initial and boundary conditions

$$u = 0, T = T_\infty, C = C_\infty, \quad t \le 0, \ \forall \ y \quad (4)$$

$$u = u_0, T = T_\infty + (T_w - T_\infty)At, C = C_\infty + (C_w - C_\infty)At, \quad y = 0 \quad (5)$$

$$u \to 0, T \to T_\infty, C \to C_\infty, \quad y \to \infty \quad (6)$$

where, $A = \frac{u_0^2}{\nu}$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^*\sigma(T_\infty^4 - T^4) \quad (7)$$

Considering the temperature difference within the flow is sufficiently small, T^4 can be expressed as the linear function. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad (8)$$

Using equations (7) and (8), equation (2) reduces to

$$\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} + 16a^*\sigma(T_\infty^3 - T) \quad (9)$$

Introducing the following non- dimensional quantities:

$$u^* = \frac{u}{u_0}, y^* = \frac{yu_0}{\nu}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, C^* = \frac{C - C_\infty}{C_w - C_\infty}, \mu = \rho\nu, t^* = \frac{tu_0^2}{\nu}$$

Making use of non-dimensional variables, the equations (1), (2) and (9) leads to (dropping asterisks)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \left(M^2 + \frac{1}{D}\right)u + Gr\,\theta + Gm\,C \quad (10)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} - \frac{R}{P_r}\theta \quad (11)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \quad (12)$$

With initial and boundary conditions

$$u = 0, \theta = 0, C = 0, \quad t \le 0, \ \forall \ y \quad (13)$$

$$u = 1, \theta = t, C = t, \quad y = 0 \quad (14)$$

$$u \to 0, \theta \to 0,\ C \to 0, \ \ as \quad y \to \infty \tag{15}$$

Where,

$R = \dfrac{16a^{*}\nu^{2}\sigma T_{\infty}^{3}}{ku_{0}^{2}}$ is the Radiation parameter,

$M^{2} = \dfrac{\sigma_{e} B_{0}^{2} \nu}{\rho u_{0}^{2}}$ is the Hartmann number,

$D = \dfrac{k u_{0}^{2}}{\nu^{2}}$ is the Darcy parameter,

$Gr = \dfrac{g\beta\nu(T_{w} - T_{\infty})}{u_{0}^{3}}$ is the thermal Grashof number,

$Gm = \dfrac{g\beta^{*}\nu(C_{w} - C_{\infty})}{u_{0}^{3}}$ tis the mass Grashof number,

$\mathrm{Pr} = \dfrac{\mu C_{p}}{k}$ is Prandtl parameter and

$Sc = \dfrac{\nu}{D_{1}}$ is the Schmidt number.

The dimensionless governing equations (10) to (12), subject to the boundary conditions (13) to (15), are solved by the usual Laplace transform technique. With help of Hetnarski's [35] has also been taken. The solutions derived are given below. Transforming equation (12) we get,

$$s\overline{C}(y,s) - C(y,0) = \frac{1}{Sc}\frac{d^{2}\overline{C}}{dy^{2}} \tag{16}$$

Using boundary conditions (13) to (15), we have,

$$\frac{d^{2}\overline{C}}{dy^{2}} - s\,Sc\,\overline{C}(y,s) = 0 \tag{17}$$

The solution of the equation (16) is

$$\overline{C}(y,s) = Ae^{\sqrt{sSc}\,y} + B\,e^{-\sqrt{sSc}\,y} \tag{18}$$

where *A* and *B* are arbitrary constants.

Again using above boundary conditions (14) and (15), we get,

$$\overline{C}(y,s) = \frac{1}{s^{2}} e^{-\sqrt{sSc}\,y} \tag{19}$$

Taking inverse Laplace transform for the equation (19) from Campbell and Foster [36] and Spiegel.M.R.[37], we obtain

$$C(y,t) = t\left(1 + 2\left(\frac{y}{2\sqrt{t}}\right)^{2} Sc\right) erfc\left(\left(\frac{y}{2\sqrt{t}}\right)\sqrt{Sc}\right) - \frac{2\left(\frac{y}{2\sqrt{t}}\right)\sqrt{Sc}}{\sqrt{\pi}}\, e^{-\left(\frac{y}{2\sqrt{t}}\right)^{2} Sc} \tag{20}$$

Also transforming equation (11);

$$s\overline{\theta}(y,s) - \theta(y,0) = \frac{1}{\mathrm{Pr}}\frac{d^{2}\overline{\theta}}{dy^{2}} - \frac{R}{\mathrm{Pr}}\overline{\theta}(y,s) \tag{21}$$

Using boundary conditions (13) and (14), it reduces to:

$$\frac{d^{2}\overline{\theta}}{dy^{2}} - (R + s\,\mathrm{Pr})\overline{\theta}(y,s) = 0 \tag{22}$$

The solution of this equation (21)

$$\overline{\theta}(y,s) = C\,e^{y\sqrt{R + s\,\mathrm{Pr}}} + E\,e^{-y\sqrt{R + s\,\mathrm{Pr}}} \tag{23},$$

where, *C* and *E* are arbitrary constants. Values of *C* and *E* can be computed using (14) and (15), we obtain

$$\overline{\theta}(y,s) = \frac{1}{s^{2}} e^{-y\sqrt{\mathrm{Pr}\left(s + \frac{R}{\mathrm{Pr}}\right)}} \tag{24}$$

Taking inverse Laplace transform for the equation (24), we obtain

$$\theta(y,t) = \frac{t}{2}\left(a_{1}e^{2\xi\sqrt{Rt}}\, erfc\,(\xi\sqrt{Pr} + \sqrt{ct}) + a_{2}e^{-2\xi\sqrt{Rt}}\, erfc\,(\xi\sqrt{Pr} - \sqrt{ct})\right) \tag{25}$$

Similarly, again taking the Laplace transform to the equation (10) and making use of the initial and boundary conditions (13) to (15), it reduces to

$$\frac{d^{2}\overline{u}}{dy^{2}} - \left[s + \left(M^{2} + \frac{1}{D}\right)\right]\overline{u}(y,s) = -GrL\{\theta(y,t)\} - GmL\{C(y,t)\} \tag{26}$$

The solution of the equation (26) is

$$\overline{u}(y,s) = F\,e^{y\sqrt{s + \left(M^{2} + \frac{1}{D}\right)}} + G\,e^{-y\sqrt{s + \left(M^{2} + \frac{1}{D}\right)}} + \frac{Gr}{1 - Pr}\frac{e^{-y\sqrt{Pr}\sqrt{s + \frac{R}{Pr}}}}{s^{2}(s - a_{3})} + \frac{Gm}{1 - Sc}\frac{e^{-y\sqrt{sSc}}}{s^{2}(s - a_{4})} \tag{27}$$

Applying the boundary conditions (14) and (15) for (26), we obtain

$$\bar{u}(y,s)=\frac{1}{s}e^{-y\sqrt{s+\left(M^2+\frac{1}{D}\right)}}+\frac{Gr}{1-Pr}\left(\frac{e^{-y\sqrt{Pr}\sqrt{s+\frac{R}{Pr}}}-e^{-y\sqrt{s+\left(M^2+\frac{1}{D}\right)}}}{s^2(s-a_3)}\right)+\frac{Gm}{1-Sc}\left(\frac{e^{-y\sqrt{sSc}}-e^{-y\sqrt{s+\left(M^2+\frac{1}{D}\right)}}}{s^2(s-a_4)}\right) \tag{28}$$

Taking the inverse Laplace transform to the equation (28), we obtain the velocity as

$$\begin{aligned}
u(y,t)=&\,a_5\,e^{-y\sqrt{\left(M^2+\frac{1}{D}\right)}}\,erfc\left(\xi-\sqrt{\left(M^2+\frac{1}{D}\right)t}\right)+a_6\,e^{y\sqrt{\left(M^2+\frac{1}{D}\right)}}\,erfc\left(\xi+\sqrt{\left(M^2+\frac{1}{D}\right)t}\right)+\\
&-\left[e^{-y\sqrt{\Pr(a_3+(R/\Pr))}}\,erfc(\xi\sqrt{\Pr}-\sqrt{(a_3+(R/\Pr))t})+e^{y\sqrt{Pr(a_3+(R/Pr))}}\,erfc(\xi\sqrt{Pr}+\sqrt{(a_3+(R/Pr))t})\right]\\
&\frac{a_{11}}{2}e^{a_3t}-\left(a_7\,e^{-y\sqrt{\Pr(R/\Pr)}}\,erfc(\xi\sqrt{\Pr}-\sqrt{(R/\Pr)t})+a_8\,e^{y\sqrt{\Pr(R/\Pr)}}\,erfc(\xi\sqrt{\Pr}+\sqrt{(R/\Pr)t})\right)\\
&-\left[e^{-y\sqrt{Pr\left(M^2+\frac{1}{D}+a_3\right)}}\,erfc\left(\xi\sqrt{Pr}-\sqrt{Pr\left(M^2+\frac{1}{D}+a_3\right)t}\right)+\right.\\
&\left.e^{y\sqrt{Pr\left(M^2+\frac{1}{D}+a_3\right)}}\,erfc\left(\xi\sqrt{Pr}+\sqrt{Pr\left(M^2+\frac{1}{D}+a_3\right)t}\right)\right]\frac{a_{11}}{2}e^{a_3t}\\
&+\left[e^{-y\sqrt{a_4Sc}}\,erfc(\xi\sqrt{Sc}-\sqrt{a_4t})+e^{y\sqrt{a_4Sc}}\,erfc(\xi\sqrt{Sc}+\sqrt{a_4t})\right]\frac{a_{12}}{2}e^{a_4t}\\
&-\left[e^{-y\sqrt{\left(M^2+\frac{1}{D}+a_4\right)}}\,erfc\left(\xi-\sqrt{\left(M^2+\frac{1}{D}+a_4\right)t}\right)+e^{y\sqrt{\left(M^2+\frac{1}{D}+a_4\right)}}\,erfc\left(\xi+\sqrt{\left(M^2+\frac{1}{D}+a_4\right)t}\right)\right]\frac{a_{12}}{2}e^{a_4t}\\
&-a_{12}\left[1+a_4t(1+2\xi^2Sc)erfc(\xi\sqrt{Sc})+\frac{2a_4t\xi\sqrt{Sc}}{\sqrt{\pi}}e^{-\xi^2Sc}\right]
\end{aligned} \tag{29}$$

The non-dimensional shear stress is given by

$$\tau=-\left(\frac{du}{dy}\right)_{y=0}=-\frac{1}{2\sqrt{t}}\left(\frac{du}{d\xi}\right)_{\xi=0} \tag{30}$$

The non-dimensional Nusselt number is given by

$$Nu=-\left(\frac{d\theta}{dy}\right)_{y=0}=-\frac{1}{2\sqrt{t}}\left(\frac{d\theta}{d\xi}\right)_{\xi=0} \tag{31}$$

The non-dimensional Sherwood number is given by

$$Sh=-\left(\frac{dC}{dy}\right)_{y=0}=-\frac{1}{2\sqrt{t}}\left(\frac{dC}{d\xi}\right)_{\xi=0} \tag{32}$$

3. RESULTS AND DISCUSSION

We discussed the exact analysis and are presented to investigate the combined effects of heat and mass transfer on the MHD flow of an incompressible viscous fluid bounded by loosely packed porous medium in an impulsively started vertical plate with variable heat and mass transfer. The expressions for the velocity, temperature and concentration are obtained by using Laplace transform technique and also discussed the physical behaviour of the dimensionless parameters such as Hartmann number *M*, Darcy parameter *D* (Permeability parameter), Radiation parameter *R*, thermal Grashoff number Gr, mass Grashoff number Gm, Prandtl number Pr and Schmidt number *Sc*. Figs. (2-12) have been displayed for the velocity, temperature and concentration. Skin friction, Nusselt number and Sherwood number are shown in Tables 1-3. The velocity, temperature and concentration profiles for some realistic values of Prandtl number Pr (Pr = 0.71, 0.16, 3 for the saturated liquid Freon at 273.3° and Pr = 7 for water) and Schmidt number Sc (Sc = 0.2 for hydrogen) respectively. From Fig. 2, this presents the velocity profile for different values of M being other parameters fixed. We noticed that the velocity decreases with increasing the Hartmann number *M*. It is due to the fact that the application of transverse magnetic field results a resistive type force (Lorentz force) similar to drag force and upon increasing the intensity of the magnetic field which leads to the deceleration of the flow. Fig. 3 is sketched in order to explore the variations of permeability parameter *D*. It is found that the magnitude of the velocity increases with increasing the values of permeability parameter *D*. This is due to the fact that increasing the permeability reduces the drag force which assists the fluid considerably to move fast. Likewise the magnitude of the velocity *u* reduced continuously with increasing the radiation parameter R from Fig. 4. The variation of velocity for different values of dimensionless time *t* and Prandtl number Pr is shown in Figs. 5 and 6. It is noticed that velocity increases with increasing time *t*. It is also observed from the Fig. 6 that the magnitude of the velocity *u* decreases with increasing Prandtl number Pr. It is clear from Fig. 7, the velocity decreases with increasing the thermal Grashof number Gr (cooling plate), where as there sharp enhancement in velocity for heating the plate, this is increase sustains away from the plate. Fig. 8 reveals that the magnitude of the velocity increases with increasing mass Grashoff number Gm throughout the fluid region. Similarly the same phenomenon is observed with increasing Schmidt number *Sc* from Fig. 9. The effect of radiation parameter *R* on the temperature profile is shown in Fig. 10. It is found that the temperatures, being as decreasing function of *R*, decelerates the fluid flow and reduce the fluid velocity. Such an effect may also be expected, here as increasing radiation parameter *R* makes the fluid thick and ultimately causes the temperature and thermal boundary layer thickness to reduce. Hence it is observed that the temperature decreases with increasing the radiation parameter *R* throughout the fluid region. The Prandtl number actually describes the relationship between momentum diffusivity and thermal diffusivity and hence control the relative thickness of the momentum and thermal boundary layers. From Fig. 11, we observed that the temperature reduces with increasing the values of Prandtl number Pr, it is also observed that the thermal boundary layer thickness is maximum near the plate and reduces with increasing distances from leading edge and finally approaches to zero. It is also justified due to the fact that thermal conductivity of the fluid decreases with increasing Prandtl number Pr and hence decreases the thermal boundary layer and the temperature profile. Fig. 12 depicts the increasing values of Schmidt number *Sc* lead to fall the concentration profiles throughout the fluid.

The numerical values of the skin friction (τ), Nusselt number (*Nu*) and Sherwood number (*Sh*) are computed and are tabulated in the Tables 1-3, in all these tables the comparison of each parameter is made with first row in the corresponding Table. It found from Table (1), the effect of each parameter on the skin friction shows that, τ enhances with increasing *R*, *D*, Pr, Gr, Gm, *Sc* and time *t*, while decreases with *M* and –Gr. It is observed from Table (2) that Nusselt number *Nu* increases with increasing *R*, Pr and *t*. From Table 3 we observed that Sherwood number goes on increasing with increasing *Sc* and *t*.

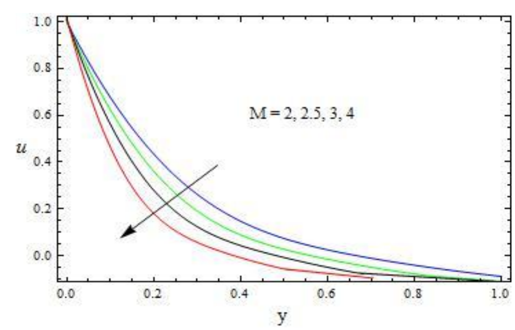

Fig. 2. The velocity profile for *u* against *M* with D=1; P= 0.71; t=0.1; Sc=2; R=1; Gr=5; Gm=10

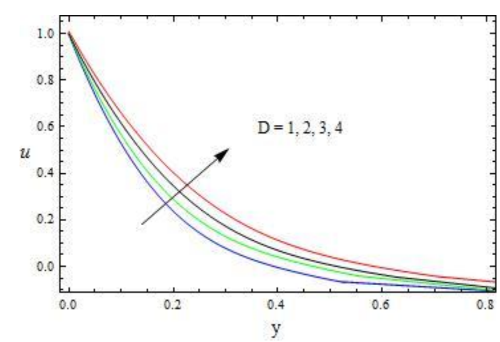

Fig. 3. The velocity Profile for *u* against *D* with M=2; P= 0.71; t=0.1; Sc=2; R=1; Gr=5; Gm=10

Table 1. The effects of various parameters on skin friction (shear stress (τ))

R	M	K	Pr	Gr	Gm	Sc	*t*	τ
1	**2**	**1**	**0.71**	**5**	**10**	**2**	**0.2**	3.67042
2	2	1	0.71	5	10	2	0.2	4.29354
3	2	1	0.71	5	10	2	0.2	4.84244
1	**3**	1	0.71	5	10	2	0.2	3.48338
1	**4**	1	0.71	5	10	2	0.2	2.15628
1	2	**2**	0.71	5	10	2	0.2	3.70531
1	2	**3**	0.71	5	10	2	0.2	3.71552
1	2	1	**0.16**	5	10	2	0.2	3.14544
1	2	1	**3**	5	10	2	0.2	5.22357
1	2	1	**7**	5	10	2	0.2	10.4094
1	2	1	0.71	**10**	10	2	0.2	3.92357
1	2	1	0.71	**15**	10	2	0.2	4.17671
1	2	1	0.71	**-10**	10	2	0.2	2.91099
1	2	1	0.71	**-15**	10	2	0.2	2.65785
1	2	1	0.71	5	**5**	2	0.2	1.96178
1	2	1	0.71	5	**15**	2	0.2	5.37906
1	2	1	0.71	5	**20**	2	0.2	7.08770
1	2	1	0.71	5	10	**3**	0.2	3.78256
1	2	1	0.71	5	10	**4**	0.2	4.36933
1	2	1	0.71	5	10	**5**	0.2	4.99703
1	2	1	0.71	5	10	2	**0.3**	4.84466
1	2	1	0.71	5	10	2	**0.4**	6.06165
1	2	1	0.71	5	10	2	**0.5**	7.04960

Table 2. The effects of various parameters on the rate of heat transfer (Nu)

R	Pr	*t*	*Nu*
1	**0.71**	**0.1**	0.195870
2	0.71	0.1	0.216376
3	0.71	0.1	0.235839
4	0.71	0.1	0.254358
1	**0.16**	0.1	0.107555
1	**3**	0.1	0.634710
1	**7**	0.1	1.393160
1	0.71	**0.2**	0.331442
1	0.71	**0.3**	0.461249
1	0.71	**0.4**	0.588593

Table 3. The effects of various parameters on the sherwood number (Sh)

Sc	*t*	*Sh*
2	**0.1**	0.104512
3	0.1	0.226218
4	0.1	0.356825
5	0.1	0.493120
2	**0.2**	0.147802
2	**0.3**	0.181019
2	**0.4**	0.209023
2	**0.5**	0.233695

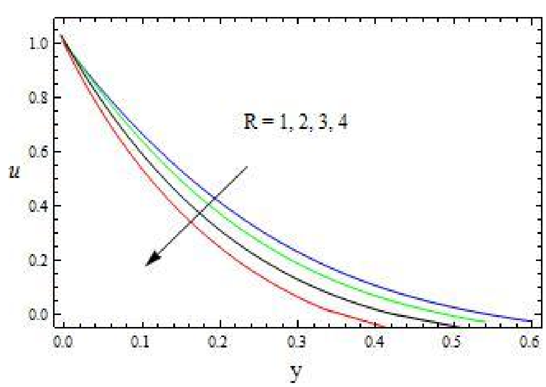

Fig. 4. The velocity profile for *u* against *R* with D=1; P= 0.71; t=0.1; Sc=2; M=2; Gr=5; Gm=10

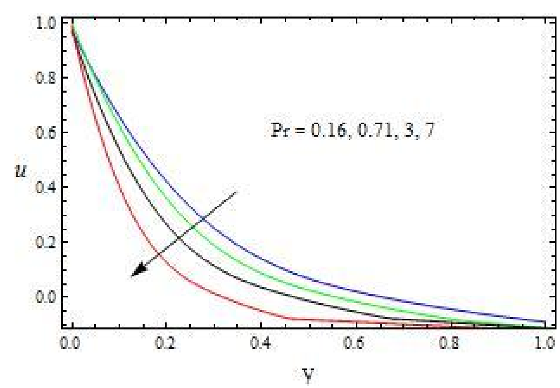

Fig. 5. The velocity profile for *u* against Pr and t with M=2; D=1; t=0.1; Sc=2; R=1; Gr=5; Gm=10

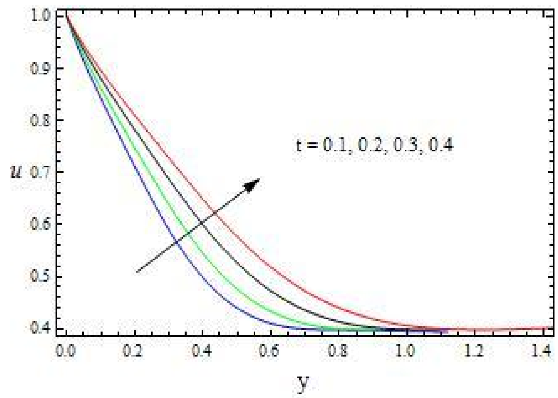

Fig. 6. The velocity profile for *u* against t with M=2; D=1; t=0.1; Sc=2; R=1; Gr=5; Gm=10

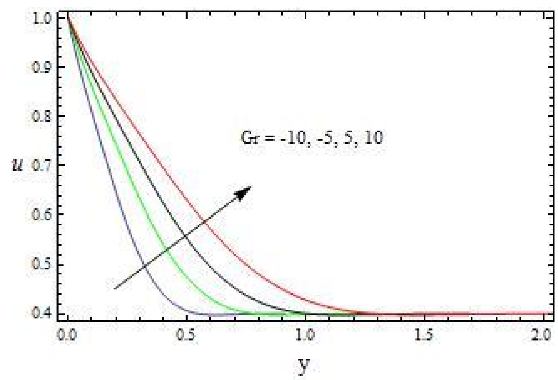

Fig. 7. The velocity profile for *u* against Gr with M=2; D=1; P=0.71; Sc=2; R=1; t=0.1; Gm=10

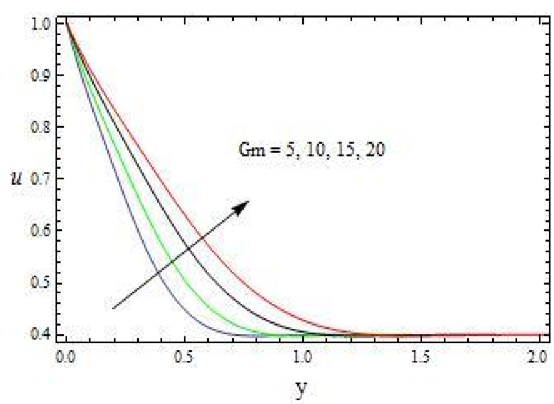

Fig. 8. The velocity profile for *u* against Gm with M=2; D=1; P=0.71, Sc=2; R=1; t=0.1; Gr=5

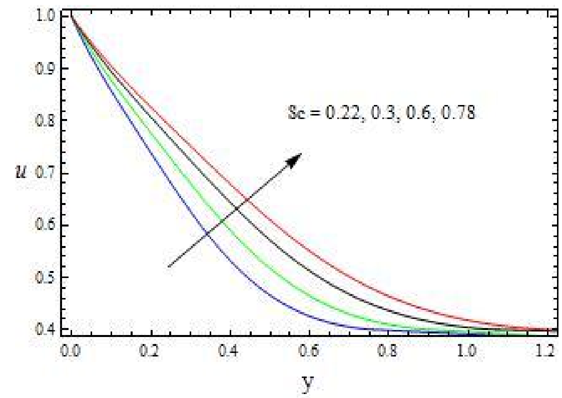

Fig. 9. The velocity profile for *u* against Sc with M=2; D=1; P=0.71, R=1; t=0.1; Gr=5; Gm=10

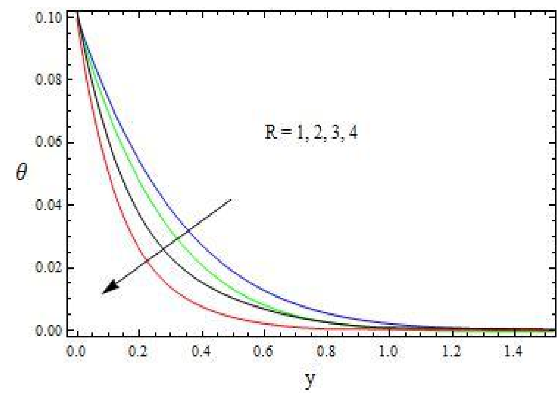

Fig. 10. The temperature profile for *θ* against R with P=0.71; t=0.1

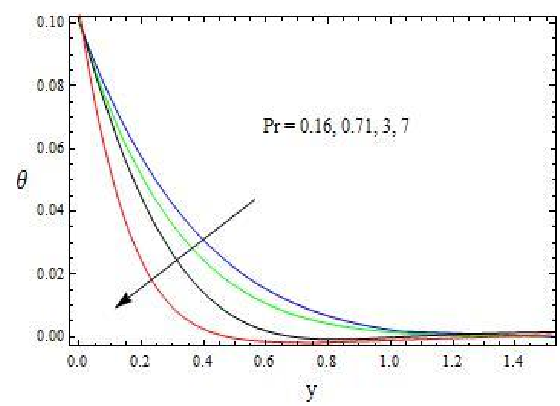

Fig. 11. The temperature profile for *θ* against Pr with R=2; t=0.1

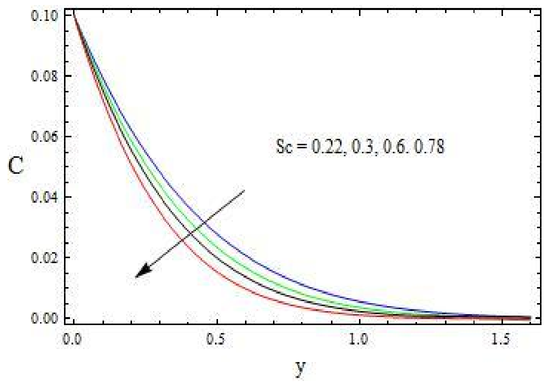

Fig. 12. The concentration profile for C against Sc with t=0.1

4. CONCLUSIONS

We have studied the unsteady MHD flow of an incompressible fluid through a loosely packed porous medium in an impulsively started vertical plate with variable heat and mass transfer. The conclusions are made as following.

1. The velocity decreases with increasing the Hartmann number M.
2. The magnitude of the velocity increases with increasing the values of permeability parameter D.
3. The magnitude of the velocity *u* enhances and reduced continuously with increasing the radiation parameter R.
4. The velocity increases with increasing time *t*. It is also observed that the magnitude of the velocity *u* decreases with increasing Prandtl number Pr.
5. The velocity decreases with increasing the thermal Grashof number Gr (cooling plate), whereas there sharp enhancement in velocity for heating the plate, this is increase sustains away from the plate.
6. The magnitude of the velocity increases with increasing mass Grashof number Gm throughout the fluid region. The same phenomenon is observed with increasing Schmidt number Sc.
7. The temperature decreases with increasing the radiation parameter R or Pr.
8. The increasing values of Schmidt number Sc lead to fall the concentration profiles throughout the fluid.
9. The skin friction enhances with increasing R, D, Pr, Gr, Gm, Sc and time *t*, while decreases with M and –Gr.
10. Nusselt number *Nu* increases with increasing R, Pr and *t.*
11. Sherwood number *Sh* goes on increasing with increasing Sc and *t.*

ACKNOWLEDGEMENTS

The author is thankful to Prof. Y. Narasimhulu, Vice-Chanceller, Rayalaseema University, Kurnool, Andhra Pradesh, India and this journal ACRI for the support to develop this document.

COMPETING INTERESTS

Author has declared that no competing interests exist.

REFERENCES

1. Soundalgekar VM. Free convection effects on the oscillatory flow an infinite, vertical porous plate with constant suction. I.Proc. R. Soc. A. 1973;333:25-36.
2. Vajravelu K, Sastri KS. Correction to Free convection effects on the oscillatory flow an infinite vertical porous plate with constant suction. I. Proc. R. Soc. A. 1977;353:221-223.
3. Soundalgekar VM, Takhar HS. On MHD flow and heat transfer over a semi-infinite plate under transverse magnetic field. Nuclear Engineering and Design. 1977;42: 233-236.
4. Soundalgekar VM, Wavre PD. Unsteady free convection flow past an infinite vertical plate with variable suction and mass transfer. Int. J. Heat Mass Transfer. 1977;20:1375-1380.
5. Soundalgekar VM, Takhar HS. Radiation effects on free convection flow past a semi-infinite vertical plate. Modelling, Measurement and Control, B. 1993;51:31-40.
6. Das UN, Deka RK, Soundalgekar VM. Effects of mass transfer on flow past an impulsively started vertical infinite vertical plate with constant heat flux and chemical reaction. Forschung in Ingenieurwesen. 1994;60:284-287.
7. Ezzat Magdy A. Magneto hydro dynamic unsteady flow of non-Newtonian fluid past an infinite porous plate. Indian J. Pure Appl. Math. 1994;25(6):655-664.
8. Raptis A, Perdikis C. Radiation and free convection flow past a moving plate. Int. J. of App. Mech. and Engg. 1999;4:817-821.
9. Muthucumara Swamy R, Ganesan P, Soundalgekar VM. Theoretical solution off

low past an impulsively started vertical plate with variable temperature and mass diffusion. Forschungim Ingenieur Wesen. 2000;66:147-151.

10. Jaiswal BS, Soundalgekar VM. Oscillating plate temperature effects on a flow past an infinite porous plate with constant suction and embedded in a porous medium. Heat and Mass Transfer. 2001;37:125-131.
11. Muthucumara Swamy R, Ganesan P. Heat transfer effects on flow past an impulsively started semi-infinite vertical plate with uniform heat flux. Nuclear Engineering and Design. 2002;215:243-250.
12. Magyari E, Pop I, Keller B. Vertical flat plate embedded in a stably stratified fluid saturated porous medium. Transport in Porous Media. 2006;62:233-249.
13. Prasad VR, Reddy NB, Muthucumara Swamy R. Radiation and mass transfer effects on two-dimensional flow past an impulsively started infinite vertical plate. Int. J. Thermal Sci. 2007;46(12):1251-1258.
14. Muthucumara Swamy R, Raj M Sundar, Subramanian VSA. Unsteady flow past anaccelerated infinite vertical plate with variable temperature and uniform mass diffusion. Int. J. of Appl. Math and Mech. 2009;5(6):51-56.
15. Toki CJ. Free convection and mass transfer flow near a moving vertical porous plate: An analytical solution. Journal of Applied Mechanics, Transactions ASME. 2008;75(1). Article ID 0110141.
16. Ahmed N, Sarmah HK. Thermal radiation effect on a transient MHD flow with mass transfer past an impulsively fixed infinite vertical plate. Int. J. of Appl. Math and Mech. 2009;5(5):87–98.
17. Reddy MG, Reddy NB. Soret and Dufour effects on steady MHD free convection flow past a semi-infinite moving vertical plate in a porous medium with viscous dissipation. Int. J. of Appl. Math and Mech. 2010;6(1):1-12.
18. Das K. Exact solution of MHD free convection flow and mass transfer near a moving vertical plate in presence of thermal radiation. African Journal of Mathematical Physics. 2010;8:29–41.
19. Das K, Jana S. Heat and mass transfer effects on unsteady MHD free convection flow near a moving vertical plate in porous medium. Bulletin of Society of Mathematicians. 2010;17:15–32.
20. Prasad VR, Muthucumara Swamy R, Vasu B. Radiation and mass transfer effects on unsteady MHD free convection flow past a vertical porous plate embedded in a porous medium: a numerical study. Int. J. of Appl. Math and Mech. 2010;6(19):1-21.
21. Rajesh V. MHD effects on free convection and mass transform flow through a porous medium with variable temperature. Int. J. of Appl. Math and Mech. 2010;6(14):1-16.
22. Osman ANA, Abo-Dahab SM, Mohamed RA. Analytical solution of thermal radiation and chemical reaction effects on unsteady MHD convection through porous media with heat source/sink. Mathematical Problems in Engineering. 2011;1-18.
23. Seth GS, Ansari MS, Nandkeolyar R. MHD natural convection flow with radiative heat transfer past an impulsively moving plate with ramped wall temperature. Heat Mass and Transfer. 2011;47:551–561.
24. Srinivasacharya D, Kaladhar K. Mixed convection flow of couple stress fluid in a non-darcy porous medium with soret and dufour effects. Journal of Applied Science and Engineering. 2012;15(4):415-422.
25. Farhad Ali, Ilyas Khan, Sharidan Shafie, Norzieha Musthapa. Heat and mass transfer with free convection MHD flow past a vertical plate embedded in a porous medium. Mathematical Problems in Engineering. 2013;1-13. Available:http://dx.doi.org /10.1155/2013/346281
26. Prabhakara Rao G, Veera Krishna M. Steady hydro magnetic rotating flow of a viscous incompressible fluid through a porous medium in a parallel plate channel with radiative heat transfer. Journal of Mathematics. 2014;10(4):04-12.
27. Ahmed A, Sarki MN, Ahmad M. Radiation effects on heat and mass transfer over an exponentially accelerated infinite vertical plate with chemical reaction. Proceedings of the International Multi Conference of Engineers and Computer Scientists. 2014;2. IMECS-2014, March 12-14, 2014, Hong Kong.
28. Das S, Guchhait SK, Jana RN. Hall effects on unsteady hydro magnetic flow past an accelerated porous plate in a rotating system. Journal of Applied Fluid Mechanics. 2015;8(3):409-417.

29. Wang CA, Tan JY. The radiation effect on the thermo-magnetic convection in participating paramagnetic medium under microgravity condition. Journal of Applied Fluid Mechanics. 2015;8(3):429-438.
30. EL-Kabeir SMM, Modather M, Rashad AM. Heat and Mass transfer by unsteady natural convection over a moving vertical plate embedded in a saturated porous medium with chemical reaction, Soret and Dufour effects. Journal of Applied Fluid Mechanics. 2015;8(3):453-463.
31. Ramesh K, Devakar M. The influence of heat transfer on peristaltic transport of MHD second grade fluid through porous medium in a vertical asymmetric channel. Journal of Applied Fluid Mechanics. 2015;8(3):351-365.
32. Khamisah Jafer, Roslinda Nazar, Anuar Ishak, Ioan Pop. MHD flow and Heat transfer over stretching/shrinking sheets with external magnetic field, viscous dissipation and joule effects. The Canedian Journal of Chemical Engineering. 2012;90(5):1336-1346.
33. Ali FM, Nazar R, Arifin NM, Pop I. Effect of hall current on MHD mixed convection boundary layer flow over a stretched vertical flat plate. Meccanica. 2011;46(5): 1103-1112.
DOI: 10,0007/s1.1012-010-9371-3.
34. Abraham JP, Sparrow EM. Friction drag resulting from the simultaneous imposed motions of a freestream and its bounding surface. Int. Jour. Heat and Fluid Flow. 2005;26(2):289-295.
35. Hetnarski RB. An algorithm for generating some inverse Laplace transforms of exponential form. ZAMP. 1975;26:249-253.
36. Campbell GA, Foster RM. Fourier integrals for practical applications, D. Van Nostrand Company, Inc. New York; 1948.
37. Spiegel MR. Theory and problems of Laplace transforms. McGraw-Hill Book Company, New York; 1986.

APPENDIX

$$\xi = \frac{y}{2\sqrt{t}},\quad a_1 = 1 + \frac{\xi\, Pr}{\sqrt{Rt}},\quad a_2 = 1 - \frac{\xi\, Pr}{\sqrt{Rt}},\quad a_3 = \frac{R - \left(M^2 + \frac{1}{D}\right)}{1 - \mathrm{Pr}},\quad a_4 = \frac{\left(M^2 + \frac{1}{D}\right)}{Sc - 1}$$

$$a_5 = \frac{1}{2}\left(a_9 + a_{10}\left(t - \frac{y}{2\sqrt{M^2 + (1/D)}}\right)\right),\quad a_6 = \frac{1}{2}\left(a_9 + a_{10}\left(t + \frac{y}{2\sqrt{M^2 + (1/D)}}\right)\right)$$

$$a_7 = \frac{a_{11}}{2}\left(1 + a_3 t - \frac{y a_3 \sqrt{\mathrm{Pr}}}{2\sqrt{R/\mathrm{Pr}}}\right),\quad a_8 = \frac{a_{11}}{2}\left(1 + a_3 t + \frac{y a_3 \sqrt{\mathrm{Pr}}}{2\sqrt{R/\mathrm{Pr}}}\right),\quad a_9 = 1 + a_{11} + a_{12},$$

$$a_{10} = \frac{Gr}{a_3(1 - \mathrm{Pr})} + \frac{Gm}{a_4(1 - Sc)},\quad a_{11} = \frac{Gr}{a_3^2(1 - \mathrm{Pr})},\quad a_{12} = \frac{Gm}{a_4^2(1 - Sc)}$$

Peer-review history:
The peer review history for this paper can be accessed here:
http://sciencedomain.org/review-history/14596